NONLINEAR ACOUSTIC WAVE PROPAGATION IN TISSUE MEDIUM

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Abstract

Measurements to comply with labeling standards and FDA requirements for diagnostic equipment are performed in water, a medium readily available and easy to use. To predict the fields in tissue, an attenuating and absorbing medium, the FDA has established a derating factor of 0.3 dB/cm-MHz [1]. This model, however, is based on a simple linear model which does not take into account the nonlinearity of the medium nor does it consider the effects of diffraction. The Khokhlov-Zabolotskaya-Kuznetsov (KZK) model for nonlinear thermoviscous diffractive waves is utilized [2]. Using this model, the acoustic pressure amplitudes along the beam axis for the fundamental and the second and third harmonics are examined for a plane piston transducer of uniform excitation. Due to the presence of nonlinearities, the dissipation of energy due to attenuation, and acoustic saturation, the FDA derating underestimates the output pressure fields in tissue using simulations based on the KZK model.

Introduction

Nonlinear wave propagation plays an important role in medical ultrasound due to the high acoustic pressure amplitude levels employed. Acoustic field calibrations are typically performed in water using a wideband hydrophone to measure the output pressure along the beam axis of a transducer. However, these output fields are not representative of *in situ* exposure levels since water is a low-loss medium. Tissue exposure levels are, therefore, estimated by applying a linear derating factor of 0.3 dB/cm-MHz. The objective of this section is to evaluate the validity of water calibrations to predict *in situ* exposure using the linear derating scheme prescribed by the FDA.

Methods

The KZK nonlinear wave model using Aanonsen's numerical solution [3] is utilized to generate fields in water and tissue media as per the following three-step process:

(1) KZK simulations are performed using the acoustical properties of water to mimic actual measurement conditions (**water fields**).

(2) Linear derating of the water fields is performed to mimic the FDA *in situ* field conditions (**derated fields**).

(3) KZK simulations are performed using the acoustical properties of tissue to mimic actual *in situ* field conditions (**tissue fields**).

The first step is to describe nonlinear wave propagation using the KZK model in a water medium. The acoustical properties of water for the simulations are given by the density, the small amplitude speed of sound, and the nonlinearity parameters are $\rho = 1000$ kg/m^3, $c_o = 1480$ m/s, and $B/A = 5.0$, respectively [4]. In addition, water is a low-loss fluid whose attenuation has a quadratic dependence on frequency with A = 0.00219 dB/cm-MHz^2 [1]. The attenuation coefficient in water for the simulations at the fundamental frequency, $f_o = 2.25$ MHz, is therefore 0.011 dB/cm.

The linear derating described in step (2) is now applied to the fields of step (1). The attenuation coefficients using the FDA derating criteria for the frequencies components at 2.25, 4.50, and 6.75, are 0.675, 1.35, and 2.03 dB/cm, respectively, which result in the derated axial acoustic pressure fields based on the FDA scheme. These fields are subsequently compared to the axial fields generated in step (3) with attenuation of tissue to simulate *in situ* conditions using the KZK model. The attenuation coefficient for tissue field simulations at the fundamental frequency, 2.25 MHz, is 0.68 dB/cm with the attenuation for the higher harmonics linearly dependent on the frequency. The other acoustical parameters such as the density, speed of sound, and nonlinearity are the same as those for water such that the only variable is the source acoustic pressure p_s.

A comparison of the derated fields with tissue fields is accomplished by evaluating the peak acoustic pressure amplitudes of the diffractive maximum and last axial maximum occurring in the near-field and transition regions, respectively. The ratio of the peak acoustic pressure amplitudes for the derated, $p_{n,derated}$, and tissue, $p_{n,tissue}$, fields gives a measure of the comparison between the two quantities:

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$$p_{n,\mathrm{ratio}} = p_{n,\mathrm{derated}} / p_{n,\mathrm{tissue}} \quad n=1,2,3 \qquad (1)$$

where n denotes the frequency component with n=1 for the fundamental, n=2 for the second harmonic, and n=3 for the third harmonic. The acoustic pressure of the derated field underestimates the acoustic pressure of the tissue field when the ratio of Equation (5.1) is less than one ($p_{n,\mathrm{ratio}} < 1$). Conversely, when the ratio is greater than one ($p_{n,\mathrm{ratio}} > 1$), the acoustic pressure for the tissue fields is overestimated by the derated fields while a value of unity ($p_{n,\mathrm{ratio}} = 1$) denotes perfect agreement between the two acoustic pressure fields being compared.

To provide some insight into the rate of attenuation between the derated fields and the tissue fields, the differences in the axial location of the peak acoustic maximum are also compared. The axial shift is calculated as follows:

$$z_{n,\mathrm{shift}} = z_{n,\mathrm{tissue}} - z_{n,\mathrm{derated}} \quad n=1,2,3 \qquad (2)$$

where $z_{n,\mathrm{tissue}}$ and $z_{n,\mathrm{tissue}}$ denote the distance of the peak acoustic pressures from the source for tissue and derated fields, respectively, at the nth harmonic.

The axial acoustic pressure field distributions throughout this chapter are presented in terms of normalized acoustic pressure amplitudes. This provides a direct way to evaluate the effects of varying the acoustic pressure amplitude emitted by the transducer on the growth of harmonics components along the beam axis. Two cases, p_s = 25 and 2500 kPa, are discussed in detail in the following sections.

Linear case, p_s = 25 kPa

We start the discussion by looking at the acoustic pressure fields generated from a source transducer with a peak acoustic pressure amplitude of 25 kPa. This is referred to as the linear case since the normalized acoustic pressure amplitude of the water fields does not exhibit significant nonlinearities (Figure 1(a)).

The diffractive maximum in the near-field occurs 4.3 cm from the source with a normalized acoustic pressure amplitude of 1.93. The presence of the harmonics is significantly less in this region than the transition region where the peak acoustic pressure maxima are located 6.92 and 6.14 cm from the source with amplitudes of 0.015 and 0.00021, respectively.

The FDA linear derating applied to the water fields yields the derated field plots of Figure 1(b). The derating process has reduced the peak acoustic pressure amplitudes of all frequency components. The spatial dependence of attenuation has a greater impact further away from the source so that the axial peak acoustic pressure amplitude for the fundamental frequency component occurs closer to the source than it did in the water field calculations. In addition, the higher-frequency terms at 4.50 and 6.75 MHz undergo greater reductions in the acoustic pressure amplitudes due to the frequency dependence of attenuation.

The normalized peak acoustic pressure amplitudes in the near-field for the fundamental and the second and third harmonics are 1.38, 0.0071, and 0.00064, respectively, at axial ranges of 4.26, 4.69, and 5.02 cm while the normalized peak acoustic pressures at the last axial maxima for frequencies at 2.25, 4.50, and 6.75 MHz are 0.830, 0.0082, 0.00013, respectively, with corresponding axial ranges of 10.1, 13.2, and 14.5. Linear derating, as a result, decreases even further the level of harmonic presence with respect to the fundamental so that distortion along the beam axis is not appreciable.

The tissue fields along the beam axis are observed in Figure 1(c) when the nonlinear model is used to generate the acoustic pressure using tissue parameters. The axial plots of the tissue fields exhibit features similar to the derated fields with the spatial peak acoustic pressure of the fundamental occurring in the near-field while spatial peaks of the second and third harmonics lie in the transition region. The normalized peak acoustic pressures, in the transition region, for the higher frequency components have amplitudes of 0.0085 and 0.00014 for 4.50 and 6.75 MHz located 13.3 and 14.6 cm from the source, respectively, while the fundamental has an amplitude of 0.837 located 10.2 cm form the source.

Tables 1(a) and (b) summarize the normalized acoustic pressures at 2.25, 4.50, and 6.75 MHz in the near-field and transition regions, respectively. The last row referred as "comparison" summarizes the results for $p_{n,\mathrm{ratio}}$ and $z_{n,\mathrm{shift}}$ which are calculated based on Equations (1) and (2) and represented by the first and second numbers of the each cell, respectively, for each frequency component. In all cases, the ratios for $p_{n,\mathrm{ratio}}$ are 0.93 or greater while the axial shifts, $z_{n,\mathrm{shift}}$, are 0.1 cm or less, demonstrating that for a source pressure of 25 kPa, nonlinear distortion does not show significant disparities in the two derating schemes. This is, in part, due to the low acoustic pressure of harmonics along the beam axis such that most of the energy is maintained in the fundamental frequency. As a result, nonlinear distortion does not significantly affect the estimates for *in situ* acoustic pressure amplitudes under the FDA derating scheme.

5.1.2 High Amplitude Case, p_s = 2.5 MPa

Thus far, only the source acoustic pressure of 25 kPa has been examined. However, when the acoustic pressure from the source is increased to 2.50 MPa, a severely distorted wave is generated in water where there is a dramatic increase in the acoustic pressure of the harmonics. The spatial peak acoustic pressures for the 2.25, 4.50, and 6.75 MHz frequency components lie in the near-field as shown in Figure 2(a) with corresponding normalized amplitudes of 1.56, 0.660, and 0.378

occurring at 4.16, 4.50, and 4.59 cm from the source. The second and third harmonics experience significant harmonic content relative to the fundamental. The peak acoustic pressure in the transition region are located at axial ranges of 8.95, 9.28, and 9.33 cm and corresponding normalized amplitudes of 0.754, 0.406, and 0.233 for the fundamental and the second and third harmonics, respectively.

The normalized acoustic pressure amplitudes for the derated and tissue fields are shown in Figure 2(b) and (c), respectively. The amplitudes along with the corresponding axial positions in the near-field and last axial maxima are summarized in Tables 2(a) and (b).. The harmonic content of acoustic pressure amplitudes are significant throughout the axial beam at an acoustic source pressure of 2.50 MPa. However, the derated and tissue fields are now analyzed to compare agreement between the two fields at the high source level (Tables 2(a) and (b)). The peak acoustic pressure for the derated fields in the near-field at the fundamental is the only value that compares favorable with the acoustic pressure for the corresponding tissue fields, i.e., $p_{1,ratio}$=0.98 and $z_{1,shift}$=0.0. However, the peak acoustic pressures between the derated and tissue fields of the harmonics begin to deviate significantly in this region since the ratios for $p_{2,ratio}$ and $p_{3,ratio}$ are now 0.77 and 0.58 respectively with corresponding axial shifts of 0.10 and 0.14 cm. The derated and tissue fields are further degraded in the transition region where the last axial maxima have $p_{1,ratio}$= 0.77, $p_{2,ratio}$=0.51, and $p_{3,ratio}$=0.11. In addition, the axial shifts are also quite severe with differences of 0.4, 1.4 and 1.9 for $z_{1,shift}$, $z_{2,shift}$, and $z_{3,shift}$, respectively. It is evident that the linear derating scheme results in faster attenuation resulting an underestimation of the peak acoustic pressure amplitudes of tissue fields using tissue parameters.

Discussion

According to linear theory, as the acoustic pressure at the source is increased the acoustic pressure fields will increase proportionately. However, when dealing with high amplitude waves in a low-loss, nonlinear medium such as water, the acoustic waves will begin to saturate. As a result, there will no longer be a one-to-one correspondence between the propagated wave and the source. Rather, more energy will be transferred to the harmonic components resulting in greater waveform distortion. However, for an attenuating medium, significantly higher acoustic source pressures are necessary before the onset of acoustic saturation. In addition, the growth of harmonics is inhibited. Since attenuation losses are frequency dependent, this results in less overall attenuation than occurring in a lossless medium where more energy is transferred to higher frequencies. As a result, for an attenuating medium the fundamental component predominates. However, the FDA derating scheme [1] assumes a linear wave propagation model which was shown to be inadequate when finite amplitude waves are considered.

References

[1] AIUM, *Acoustic Output Measurement and Labeling Standard for Diagnostic Ultrasound Equipment*, Rockville MD, AIUM (1992).

[2] B.K. Novikov, O.V. Rudenko, and T.I. Timoshenko, *Nonlinear Underwater Acoustics*, New York, American Institute of Physics (1987).

[3] S.I. Aanonsen, "Numerical computation of the nearfield of a finite amplitude sound beam," Rep. No. 73, Department of Mathematics, University of Bergen (1983).

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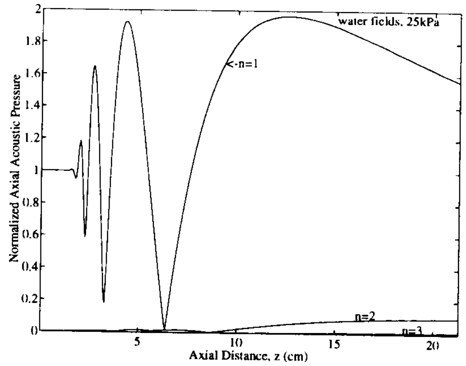
Table 1: Normalized peak acoustic pressure amplitudes for derated and tissue fields in the (a) near-field and (b) transition region at p_s=25 kPa

(a)

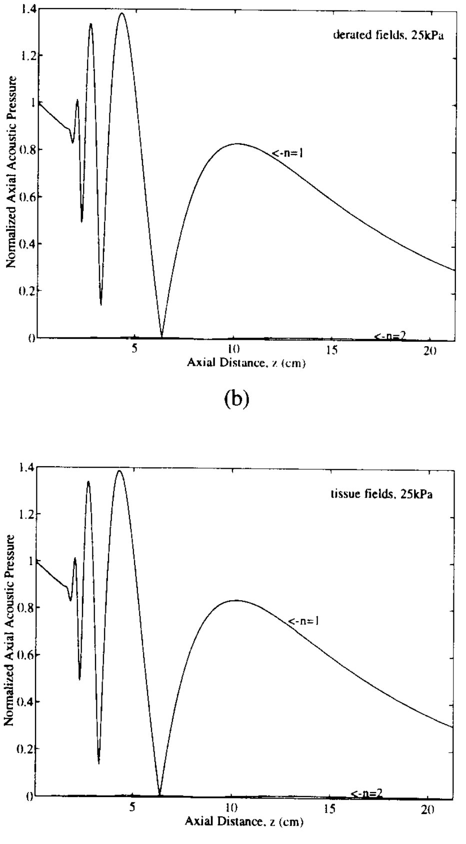
	fundamental		second harmonic		third harmonic	
	ampl	z(cm)	ampl	z(cm)	ampl	z(cm)
derated	1.38	4.26	0.0071	4.69	6.4e-5	5.02
tissue	1.39	4.26	0.0071	4.74	6.6e-5	5.03
compare	0.99	0.0	0.99	0.05	0.97	.01

(b)

	fundamental		second harmonic		third harmonic	
	ampl	z(cm)	ampl	z(cm)	ampl	z(cm)
derated	0.830	10.1	0.0082	13.2	0.00013	14.5
tissue	0.837	10.2	0.0085	13.3	0.00014	14.6
compare	0.99	0.1	0.96	0.1	0.93	0.1

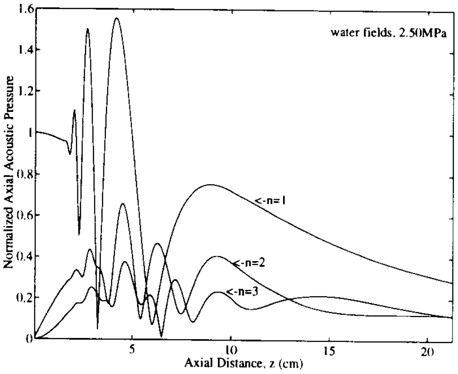


(a)

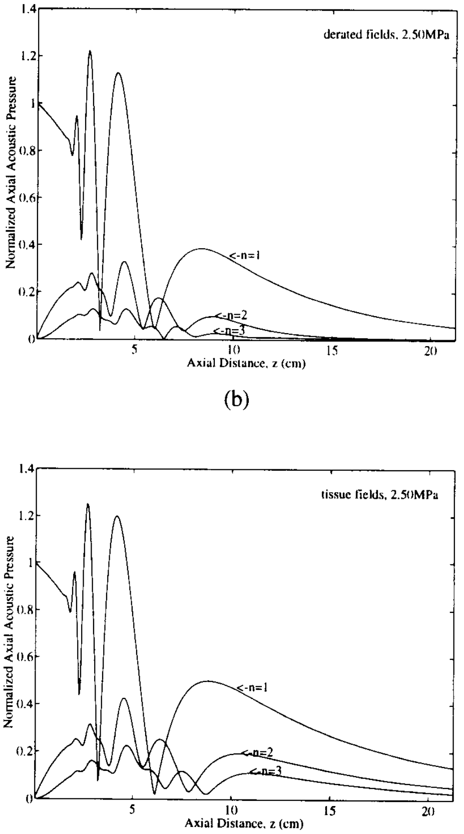


(c)

Figure 1: Normalized axial acoustic pressure amplitude for (a) water fields (b) derated fields (c) tissue fields at frequencies 2.25 (n=1), 4.50 (n=2), and 6.75 MHz (n=3) when p_S = 25 kPa.



(a)



(c)

Figure 2: Normalized axial acoustic pressure amplitude for (a) water fields (b) derated fields (c) tissue fields at frequencies 2.25 (n=1), 4.50 (n=2), and 6.75 MHz (n=3) when p_S = 2.50 MPa.

Table 2: Normalized peak acoustic pressure amplitudes for derated and tissue fields in the (a) near-field and (b) transition region at p_S=2.50 MPa

(a)

	fundamental		second harmonic		third harmonic	
	ampl	z(cm)	ampl	z(cm)	ampl	z(cm)
derated	1.22	2.66	0.329	4.45	0.130	4.55
tissue	1.25	2.66	0.426	4.55	0.226	4.69
compare	0.98	0.0	0.77	0.10	0.58	0.14

(b)

	fundamental		second harmonic		third harmonic	
	ampl	z(cm)	ampl	z(cm)	ampl	z(cm)
derated	0.386	8.37	0.099	8.95	0.012	9.1
tissue	0.500	8.80	0.19	10.3	0.113	11.0
compare	0.77	0.43	0.51	1.4	0.11	1.9