ESTIMATING THE MEDICAL ULTRASOUND *IN VIVO* POWER SPECTRUM

BY

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ABSTRACT

This thesis considered the estimation of the *in vivo* power spectrum from the backscattered waveforms by finding the total attenuation along the propagation path. The total attenuation was estimated by assuming model for the scatterers (i.e., spherically symmetric Gaussian impedance distributions of unknown size) and then solving for the size and total attenuation simultaneously from the frequency dependence of the backscattered spectrum. The attenuation and scatterer size could be accurately and precisely estimated provided that sufficient frequency data was available. The accuracy and precision were significantly improved by increasing the range of frequencies used in the estimate. In addition, some improvement could be obtained by increasing the length of the window used to gate the backscattered RF echoes (i.e., samples in frequency domain independent). Although only applied to the estimation of scatterer size in this thesis, the estimation of the in vivo power spectrum using the developed methods could be applied to other tissue characterization procedures as well as estimating the temperature increase in the tissue from ultrasound exposures.

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LIST OF SYMBOLS

a = aperture radius for a spherically focused source.

A = term for form factor written as a power law.

A_{comp} = generalized attenuation-compensation function including focusing effects along the beam axis.

a_{eff} = effective radius of scatterer.

$a_{eff\ j}$ = estimated effective radius of scatterer found from one set (i.e. 25 averaged RF echos) of simulated backscatter waveforms.

$\overline{a}_{eff}$ = mean value of estimated effective radius from all sets of backscattered waveforms (i.e., $\overline{a}_{eff} = \sum_{\forall j} a_{eff\ j} \Big/ \sum_{\forall j} j$).

A_{OO} = Oelze-O'Brien attenuation-compensation function.

A_{OM} = O'Donnell-Miller attenuation-compensation function.

A_{PC} = point attenuation-compensation function.

a_{real} = real value for effective radius of scatterer when comparing to estimated value.

ASD = average squared difference value used when solving minimization.

ASD_{plane} = average squared difference between theory and measurement for Plexiglas experiment.

A_{ν} = affinities associated with vibrational mode of ν-type molecules in fluid particle.

b_{γ} = correlation function of individual scatterer.

B_{γ} = correlation function related to field and scatterers.

c = effective small-signal sound speed of medium.

C_1, C_2 = constants used in derivations.

c_n = small-signal sound speed of region n.

c_o = small-signal sound speed of water.

c_T = speed of sound assuming isothermal propagation.

c_V = the specific heat at constant volume.

c_{Vfr} = the specific heat at constant volume when vibrational modes are not allowed.

$c_{V\ eff}$ = the effective specific heat at constant volume.

$c_{\nu\nu}$ = specific heat at constant volume contribution from vibrational mode of ν-type molecules in fluid particle.

d = characteristic length describing Gaussian impedance distribution.

$d\widetilde{k}_n$ = difference between the effective complex wavenumber along the propagation path and the complex wavenumber in a particular region (i.e., $d\widetilde{k}_n = \widetilde{k} - \widetilde{k}_n$).

e = thermodynamic internal energy.

E = total energy of thermodynamic system.

E[], E_N[] = expected value of term in brackets.

$\hat{e}_j$ = unit vector defining coordinate system in thermodynamic calculations (j = 1, 2, or 3).

e_ν = thermodynamic internal energy associated with vibrational mode of ν-type molecules in fluid particle.

f = frequency.

F = focal length for a spherically focused source.

$f\#$ = f-number for a spherically focused source (i.e., $f\# = F/2a$).

f_o = the frequency corresponding to the spectral peak of the Gaussian spectrum (i.e., $\exp\left(-\left(\frac{f - f_o}{\sqrt{2}\sigma_\omega}\right)^2\right)$).

$\widetilde{f}_o$ = f_o for backscattered spectrum modified by scatterer size.

$\widetilde{f}_o'$ = f_o for backscattered spectrum modified by scatterer size and attenuation along propagation path.

f_{peak} = frequency corresponding to the spectral peak at each inclination angle (i.e., $V_{plane} \propto \exp\left(-\left(\frac{f - f_{peak}}{2\sigma_{\omega p}^2}\right)^2\right)$).

f_R = the parameter used to set the location of the Rayleigh distribution along the frequency axis (i.e., $|f| \cdot \exp\left(-\left(\frac{f - f_R}{\sigma_R}\right)^2\right)$).

F_R = radiation force (i.e., $F_R = 2\alpha_{loc} I/c$).

F_γ = form factor for scatterer.

$g(\vec{r}_d, \vec{r}\,')$ = effective Green's function valid from the scattering region to the detector.

G_{corr} = windowing correction term for spectrum.

$g_n(\vec{r}, \vec{r}\,')$ = Green's function for region *n*.

G_o ,G_{o_trans} = geometric gain value on receive/transmit for pressure field at focus when W_{source} is approximated by a Gaussian (units of m).

G_{sp},F_{sp},Λ= functions/variables used to find stationary phase solution to Green's function for planarly layered medium.

$g_T(\vec{r}\,', \vec{r}_T)$ = Green's function valid from the transmitter to the scattering region.

G_T = dimensionless aperture gain function that accounts for the focusing of the ultrasound source.

g_{win} = windowing function used to gate the signal.

G_{win} = Fourier transform of $g_{win}(t)$.

g_{win2} = windowing function used for homomorphic filtering.

H = dimensionless filtering characteristics for the ultrasound source.

$H_0^{(1)}$ = 0th order Hankel function of the first kind.

I = temporal average intensity of ultrasound field.

I' = instantaneous intensity of ultrasound field.

$j_o()$ = 0th order spherical Bessel function of the first kind.

$J_o()$ = 0th order Bessel function of the first kind.

k = effective wavenumber along the propagation path.

$\tilde{k}$ = effective complex wave number along the propagation path (i.e. $\tilde{k} = k + \mathrm{i}\,\alpha$).

k_o = wavenumber in water.

k_n = wavenumber in region *n* (i.e., $k_n = \dfrac{2\pi}{\lambda_n}$).

$\tilde{k}_n$ = complex wavenumber in region *n* (i.e., $\tilde{k}_n = k_n + \mathrm{i}\,\alpha_n$).

$\tilde{k}_{nz}$ = complex wavenumber in the *z* direction in region *n* (i.e., $\tilde{k}_{nz} = \sqrt{\tilde{k}_n^2 - k_\xi^2}$).

K_{uV} = conversion constant relating voltage to particle velocity for ultrasound source (units of m/s V^{-1}).

$\tilde{k}_{zs}, k_{\xi s}$ = wavenumbers corresponding to stationary phase point.

k_ξ = wavenumber in the ξ-direction (i.e., $k_\xi = \sqrt{\tilde{k}_n^2 - \tilde{k}_{nz}^2}$).

L = total width of windowing function.

$\mathbf{M_{image}}$ = matrix used to generate image point.

$\bar{n}$ = average scatterer number density.

$\hat{n}$ = the outward unit normal on surface of the fluid particle.

N = last number in set of indices.

$N(f)$ = additive electronic noise.

N_{dB} = minimum value allowed for N_{floor} when no electronic noise has been added.

$\vec{n}_f$ = the outward normal for the plane at arbitrary angle to beam axis.

N_{Floor} = noise floor of system used when selecting usable frequencies.

$\vec{n}_I$ = vector perpendicular to the aperture plane of the image source.

N_{lines} = number RF echoes used when determining an estimate for P_{scat}.

p = pressure.

p' = small perturbation to ambient pressure.

p_{inc} = pressure field incident on the scatterers.

$P_n, P_p, \overline{P}_n$ = terms used to fit Gaussian distribution to spectrum in log domain.

p_o = ambient pressure.

p_{plane} = pressure field from rigid plane placed near focal plane.

P_{ref} =reference spectrum (i.e., $P_{ref}(f) = k_o^4 |V_{inc}(\omega)|^2 |H(\omega)|^4$).

p_s = scattered pressure field.

$P_{scat} = E\left[|V_{refl}|^2\right]$ estimated from set of waveforms.

p_{tot} = total pressure field (i.e., $p_{tot} = p_s + p_{inc}$).

$\dot{\vec{q}}$ = the heat flow across the boundary of the fluid particle.

$\dot{q}_{blood}$ = the heat removed by blood perfusion.

$\dot{Q}_e$ = the rate heat flows into the thermodynamic system

$\dot{q}_i$ = the heat generated within the fluid particle.

$\dot{Q}_i$ = the rate heat is generated or removed internally for a thermodynamic system.

$\dot{q}_{source}$ = the heat generated by ultrasound source.

$\vec{r}, \vec{r}\,', \vec{r}\,''$ = spatial locations in spherical coordinates.

$\Delta\vec{r}, \vec{s}$ = change of spatial variables (i.e., $\Delta\vec{r} = \vec{r}\,' - \vec{r}\,''$ and $\vec{s} = (\vec{r}\,' + \vec{r}\,'')/2$).

$\vec{r}_f$ = locations on rigid reference plane in spherical coordinates.

$\vec{r}_I$ = points on aperture plane of image source.

r_{max} = maximum distance off of beam axis used when comparing fields.

$\vec{r}_n$ = location of single scatterer.

$\vec{r}_s$ = location of point source in spherical coordinates.

$\vec{r}_T, \vec{r}_d$ = locations on aperture plane of transmitter/detector in spherical coordinates.

$R_{\gamma\gamma}$ = autocorrelation function for the scatterer.

$\Re_{\gamma\gamma}$ = power spectral density function for the scatterer.

r'_ρ = distance off of beam axis.

s = entropy.

S = strain tensor.

$\dot{S}$ = time derivative of S.

S^* = surface of single fluid particle.

S_f = rigid plane near focal plane used to acquire reference waveform.

s_{fr} = entropy associated with translational and rotational motions of fluid particle.

S_I = aperture plane of image source.

SNR = signal-to-noise ratio.

S_T = aperture plane of ultrasound transmitter.

s_θ = variable used in substitution when evaluating integral.

s_v = entropy associated with vibrational motions of fluid particle.

t = time.

T = temperature.

T' = small perturbation to ambient temperature.

T_c = ambient temperature.

T_{cep} = value used to set the amount of homomorphic filtering.

T_{eff} = effective temperature.

$Term_\xi, Term_{a_{eff}}, Term_\alpha, Term_{\alpha,a_{eff}}$ = terms describing ASD surface for Spectral Fit algorithm.

T_{ij} = transmission coefficient from region i to region j.

T_o = product of all transmission coefficients.

T_{win} = total width of windowing function applied to time-domain waveform (i.e., $T_{win} = 2L/c$).

T_ν = temperature associated with vibrational mode of ν-type molecules in fluid particle.

$\vec{u}$ = particle velocity.

u_z = particle velocity perpendicular to aperture plane of ultrasound transmitter/detector.

V' = volume containing scatterers contributing to the scattered signal.

V^* = volume of single fluid particle.

V_{cep_i} = a RF echo expressed in cepstrum domain.

V_{inc} = voltage applied to the ultrasound source during transmit.

V_j = backscattered voltage spectrum for a single RF echo.

v_{noise} = example noise signal voltage in time domain (i.e., no signal transmitted by source).

V_{plane} = voltage from ultrasound source due to the backscatter from rigid plane near focus.

$V_{measured}$ = voltage spectrum returned from Plexiglas experiment.

V_{refl} = voltage spectrum from ultrasound source due to the backscatter from scatterers.

v_{refl_i} = voltage of a RF echo in time domain.

V_s = average scatterer volume.

V_{theory} = theoretical voltage spectrum for Plexiglas experiment.

w = energy per unit volume.

$\dot{W}$ = the rate work interacts with a thermodynamic system.

W_{source} = term describing fall off of field in focal region (units of m^2).

$\hat{W}_{source}$ = magnitude of W_{source}.

w_x, w_y, w_z = equivalent Gaussian dimensions on receive of pressure field in focal region.

w_{xo}, w_{yo} = equivalent Gaussian beamwidths at center frequency of transducer.

$w_{x\_trans}, w_{y\_trans}, w_{z\_trans}$ = equivalent Gaussian dimensions on transmit of pressure field in focal region.

w_{zm} w_{zb} = linear fit parameters for w_z (i.e., $w_z = w_{zm} \cdot \lambda + w_{zb}$).

$X, \overline{X}$ = terms used in minimization scheme to solve for scatterer size.

x_I, y_I, z_I = coordinate location of image point.

z, ξ = Cartesian coordinate system for planarly layered medium (i.e., $\xi = \sqrt{x^2 + y^2}$).

$\hat{z}, \hat{\xi}$ = unit vectors defining z, ξ axis.

z_j = location of region boundaries in planarly layered medium (j = 1,2,3, ...).

z_f = distance of rigid plane to the focal plane.

z_o = offset in window placement due to errors in sound speed.

z_{oF} = shift of the focus away from geometric focus at a particular frequency.

z_p = distance from the focus that the beam axis intersects with the inclined plane.

z_T, z_d = distance of aperture plane of the ultrasound transmitter/detector to the focal plane.

z_{trans} = distance from transmit focus to receive focus.

α = effective attenuation along the propagation path.

α_b = intercept term of attenuation assuming general linear frequency dependence (i.e., $\alpha = \alpha_o f + \alpha_b$).

α_{error} = error in attenuation associated with inclination angle of plane.

α_{loc} = local absorption coefficient of medium.

α_n = attenuation in region n.

α_o = slope of attenuation assuming strict linear frequency dependence (i.e., $\alpha = \alpha_o \cdot f$).

$(\alpha_o z_T)_j$ = estimated attenuation along the propagation path for single data set.

$\overline{(\alpha_o z_T)}$ = mean value for attenuation along the propagation path from all sets of backscattered waveforms (i.e., $\overline{(\alpha_o z_T)} = \sum_{\forall j} (\alpha_o z_T)_j \Big/ \sum_{\forall j} j$).

α_{real} = real value for attenuation along the propagation path when comparing to estimated value.

β_{therm} = the coefficient of thermal expansion.

γ = combined perturbation of density and compressibility (i.e., $\gamma(\vec{r}) = \gamma_\kappa(\vec{r}) - \gamma_\rho(\vec{r})$).

γ_{max} = largest value of γ for Gaussian impedance distribution.

γ_o^2 = mean squared variation in acoustic impedance per scatterer.

Γ_{plane} = reflection coefficient of plane.

γ_κ = local perturbation in the compressibility due to the scatterers (i.e., $\gamma_\kappa(\vec{r}) = \dfrac{\kappa_s(\vec{r}) - \kappa}{\kappa}$).

γ_ρ = local perturbation in the density due to the scatterers (i.e., $\gamma_\rho(\vec{r}) = \dfrac{\rho_s(\vec{r}) - \rho}{\rho_s(\vec{r})}$).

$\delta()$ = Dirac delta function.

δ_{ij} = Kronecker delta function.

θ_d = dilatation term (i.e., fractional increase in volume given by $\theta_d = S_{11} + S_{22} + S_{33}$).

$\dot{\theta}_d$ = time derivative of θ_d.

θ_f, ϕ_f = angles describing orientation of plane with beam axis.

κ = compressibility of background medium surrounding scatterers.

κ_s = compressibility of scatterers.

κ_t = the thermal conductivity of the medium.

λ = wavelength.

λ_L ,μ_L = Lame' constants.

λ_o= the wavelength corresponding to the spectral peak from the reference spectrum.

μ = the shear viscosity.

μ_B = the bulk viscosity.

$\vec{\xi}_d$ = particle displacement.

$\xi_s(f)$ = spectral variations due to random scatterer spacing.

ξ_x, ξ_y, ξ_z = coordinate system for the image source.

$\hat{\xi}_x, \hat{\xi}_y, \hat{\xi}_z$ = unit normal vectors defining coordinate system for the image source.

ρ = density of background medium surrounding scatterers.

ρ' = small perturbation to ambient density.

ρ_c = ambient density.

ρ_o = density of water.

ρ_n = density of region n.

ρ_s = density of scatterers.

σ = tensor representing the external forces acting on the fluid particle.

$\sigma_{a_{lower}}$ = percent deviation in values of scatterer size for sizes smaller than the mean size (i.e.,

$a_{eff\,j} < \overline{a}_{eff}$).

$\sigma_{a_{upper}}$ = percent deviation in values of scatterer size for sizes larger than the mean size (i.e., $a_{eff\,j} > \overline{a}_{eff}$).

$\sigma_{f^2,\xi}, \sigma_{f,\xi}$ = terms describing the frequency dependence of $\xi_s(f)$.

σ_g = the bandwidth term for the Gaussian distribution approximating the windowing function (i.e., $|G_{win}(f)|^2 \propto e^{-\frac{f^2}{2\sigma_g^2}}$).

σ_n = the average of the normal components of the stress tensor.

σ_R = the bandwidth term for Rayleigh distribution (i.e., $|f| \cdot \exp\left(-\left(\frac{f - f_R}{\sigma_R}\right)^2\right)$).

$\sigma_{\alpha_{lower}}$ = deviation in dB/MHz in values of attenuation for attenuations smaller than the mean attenuation (i.e., $(\alpha_o z_T)_j < \overline{(\alpha_o z_T)}$).

$\sigma_{\alpha_{upper}}$ = deviation in dB/MHz in values of attenuation for attenuations greater than the mean attenuation (i.e., $(\alpha_o z_T)_j > \overline{(\alpha_o z_T)}$).

σ_ω = the bandwidth term for Gaussian distribution (i.e., $\exp\left(-\left(\frac{f - f_o}{\sqrt{2}\sigma_\omega}\right)^2\right)$).

$\tilde{\sigma}_\omega = \sigma_\omega$ for backscattered spectrum modified by scatterer size.

$\sigma_{\omega p}$ = Gaussian bandwidth for reflected voltage from inclined plane (i.e., $V_{plane} \propto \exp\left(-\left(\frac{f - f_{peak}}{2\sigma_{\omega p}^2}\right)^2\right)$).

τ_{cep} = time values in cepstrum domain.

Φ = field term for scattered pressure field.

ϕ_{comp} = complete velocity potential field for focused source.

ϕ_{inc} = incident field term for scattered pressure field.

ϕ_{ij} = components of rate-of-shear tensor.

Φ_o = field term for reflected voltage.

Ψ = transmission term for scattered pressure field.

Ψ_o = transmission term for reflected voltage.

ω = radian frequency.

$\Omega(\omega), C_x, C_y, C_z, C_k, X_{1x}, X_{1y}, X_{1z}, X_{2x}, X_{2y}, X_{2z}, X_k, Y_{1x}, Y_{1y}, Y_{1z}, Y_{2x}, Y_{2y}, Y_{2z}, Y_k, Y'_{2z}, Y''_{2z}, Y'_k$ = grouping of terms to facilitate the derivation of the voltage returned form the inclined plane.