APPENDIX C:
THE RLC CIRCUIT MODEL FOR A PIEZOELECTRIC TRANSDUCER

In this appendix, the equivalent RLC circuit model as presented by *Bechmann and Fair* [1966] is analyzed. First, an expression for the electrical input impedance for the model will be derived in terms of the circuit elements. Then the values of the circuit elements for the air backed crystal will be provided.

Input Impedance of the RLC Circuit

The RLC circuit model for the piezoelectric transducer is shown in Figure C.1 below [*Bechmann and Fair*, 1966].
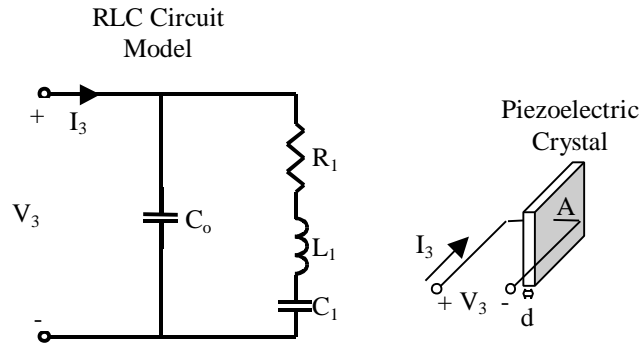

Figure C.1: The RLC circuit model for a piezoelectric transducer.

Using basic circuit analysis, the input impedance for this model is given by

$$Z_{in\_RLC} = \frac{wR_1C_1 + j(w^2L_1C_1 - 1)}{jw^2R_1C_1C_o - wC_o(w^2L_1C_1 - 1) + wC_1} \qquad \text{(C.1)}$$

Likewise, the admittance would be given by

$$Y_{in\_RLC} = jwC_o + wC_1 \frac{R_1wC_1 - j(w^2L_1C_1 - 1)}{(R_1wC_1)^2 + (w^2L_1C_1 - 1)^2} \qquad \text{(C.2)}$$

From Equation (C.2), it is clear that the real part of the admittance will achieve a maximum at the resonant frequency of the circuit.

Element Parameters in the RLC Model

      Although a complete analysis of the RLC model is beyond the scope of this thesis, some closed form expressions for the values of the circuit parameters for an air backed crystal will be provided. By including all of the relevant terms in the analysis provided by *O'Brien* [2000] it can be shown that for the air backed case the element parameters are

$$R_1 = \frac{Z_m}{4C_o^2 h^2}$$

$$L_1 = \frac{Ard}{8C_o^2 h^2} \tag{C.3}$$

$$C_1 = \frac{8C_o^2 h^2 d}{p^2 c^2 Ar - 8C_o h^2 d}$$

where

$$C_o = \frac{eA}{d} \tag{C.4}$$

as was stated in Appendix B, and Z_m is the radiation impedance of the material into which the crystal is radiating. From these expressions, it is clear that a voltage drop across R_1 would correspond to power radiated by the crystal. Before, leaving this section, it is important to point out that the circuit resonant frequency given by $w'_o = \dfrac{1}{\sqrt{L_1 C_1}}$ is close

to the crystal resonant frequency given by $w_o = \dfrac{cp}{d}$, but the two quantities are not exactly the same.