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# ANGULAR DISTRIBUTION OF SCATTERED ULTRASOUND FROM A SINGLE STEEL SPHERE IN AGAR GELATIN: A COMPARISON BETWEEN THEORY AND EXPERIMENT

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Experimental results for the angular distribution of scattered ultrasound in water from a 0.635 mm diameter steel sphere embedded in agar were obtained for interrogating frequencies of 1.0, 2.25, 3.5 and 5.0 MHz. The experimental results compared favorably with theoretical predictions based on the work of Faran. The results for both theory and experiment are presented in the form of the Differential Scattering Cross Section (DSC) for scattering angles from 170° to 44°. The implications of this study on calculating the expected scattering behavior of gelatin based ultrasound phantoms are briefly discussed.

Key words: Agar; phantom; scattering; sphere; ultrasound.

## I. INTRODUCTION

In the early 1950's, James Faran [1] developed theoretical expressions to predict the magnitude and angular distribution of sound scattered in water from solid cylinders and spheres. The theory assumed plane incident waves of a single frequency, insonifying a single scatterer surrounded by a nonviscous fluid. The expressions were extensions of earlier work [2] that did not take into account the mode conversion (to shear waves) which can occur when compressional waves impinge upon a scatterer made of solid material. To verify this theory, Faran performed a number of experiments measuring the angular distribution of sound scattered by metal cylinders and spheres and normalized them to the magnitude of the incident pressure wave amplitude. The experiments were performed at various discrete ultrasonic frequencies.

In the course of experimentally testing the theoretical expressions describing the scattering of sound from a solid sphere, Faran encountered a major experimental problem which ultimately limited his results to two studies. This problem was, in essence, how to suspend a spherical target (of approximately 0.07 inches in diameter) in water without having the suspending device perturb the measurements. Faran's solution was to glue the sphere to a thin (0.005 inch diameter) nylon thread. The technique proved difficult and the gluing substance itself did perturb the experimental results. Nonetheless, sufficient agreement was achieved between theory and experiment to verify Faran's theoretical work. Since then,

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little experimental effort has been expended to repeat Faran's work on the scattering of sound from solid spheres. This may be due to the previously described difficulty experienced in suspending the spherical target.

We will discuss a promising alternative technique for suspending a small spherical target for scatter measurements. The target, which consists of a steel sphere placed centrally in a larger agar gelatin cylinder, was constructed as follows. First the target sphere was placed on a flat surface and a small number of drops of molten agar were dropped on the sphere and allowed to congeal. At this point, the target sphere is embedded in a thin jacket of congealed agar. A thin (0.003 inch diameter) wire is laced through the agar jacket and the wire is suspended across the cavity of the plexiglass's cylindrical target mold. The suspended bead is positioned centrally in the cylindrical cavity formed by the mold. Next, the mold is filled with molten agar and allowed to solidify. During the congealing process, small air bubbles rise out of the body of the cylindrical phantom and collect on the top surface where they dissolve into the surrounding air. After the gelatin has solidified, the thin supporting wire is pulled from the gelatin and the agar is removed from the plexiglass mold. The small hole left in the agar due to the removal of the support wire, rapidly fills with unbound water from the surrounding agar and is thus invisible to the interrogating sound waves. One concern using this technqiue is that the agar provides a viscous environment for the target while Faran's theory stipulates a nonviscous environment as a precondition for applicability. Fortunately, excellent agreement between experiment and theory was achieved using this target support scheme. This level of agreement points to the applicability of Faran's theory in the case in which small metal spheres are embedded in an agar matrix. The success of this technique may lead investigators to adopt such a sphere-in-agar phantom as a standard ultrasonic scattering target for use in measuring force-to-voltage transfer properties of piezoelectric transducers [3], and in testing the correct operation of an ultrasound scattering apparatus [4]. This note will discuss in detail the experimental study that was performed to test the new suspension technique. The implication of the results will be briefly addressed in the conclusion section.

## II. DESCRIPTION OF THE STUDY

The study that was performed was essentially a comparison between theoretical predictions and experimental measurements of the distribution of the sound scattered from a small spherical ball bearing. A diagram of the experimental geometry that was used is shown in figure 1.

The tank [4] has a radius of 120 cm and a depth of 24 cm. During scatter measurements, the tank was filled to a depth of 22 cm with

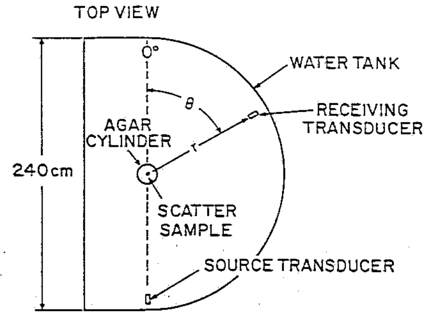

Fig. 1 Diagram of the experimental geometry used in measuring the angular distribution of scattered sound.

Table I. Relevant parameters of the source and receiver transducers. S and R stands for source and receiver respectively.

| Transducer | Nominal Frequency (MHz) | Diameter (cm) | Last Axial Maximum (cm) | Operating Distance (cm) |
|---|---|---|---|---|
| S | 1.0 | 2.86 | 13.6 | 60 |
| R | 1.0 | 1.27 | 2.7 | 60 |
| S | 2.25 | 2.54 | 24.2 | 65 |
| R | 2.25 | 0.64 | 1.51 | 60 |
| S | 3.5 | 1.91 | 21.2 | 65 |
| R | 3.5 | 0.64 | 2.35 | 60 |
| S | 5.0 | 1.27 | 13.44 | 75 |
| R | 5.0 | 0.64 | 3.36 | 65 |
| S | 6.0 | 1.27 | 16.13 | 75 |
| R | 6.0 | 0.64 | 4.03 | 65 |

distilled water. The transducers employed consisted of four pairs of Aerotech (Lewistown, PA 17044) Delta Series, nonfocused transducers. Each pair had a source and receiving transducer with matched center frequencies in the passband (to increase the efficiency of power transfer at the chosen interrogating frequency) and operated at one of the following frequencies: 1.0 MHz, 2.25 MHz, 3.5 MHz and 5.0 MHz. Additional information concerning the transducer pairs may be found in table I. The scatterer was a 0.3175 mm radius, type 440C stainless steel ball bearing (Precision Ball, Winsted Precision Ball Company, Winsted, CT 06098) that was embedded in a 4.8 cm radius agar cylinder (Fig. 2). The longitudinal speed of sound of the agar material was measured at 22°C via a time-of-flight technique [4] and found to be 1500 m/s.

It was desired to compare theory with experiment at selected frequencies. Approximations to the desired plane continuous wave situations were obtained by pulsing the source transducer with 20 to 30 microsecond sinusoidal continuous wave bursts and placing the target a

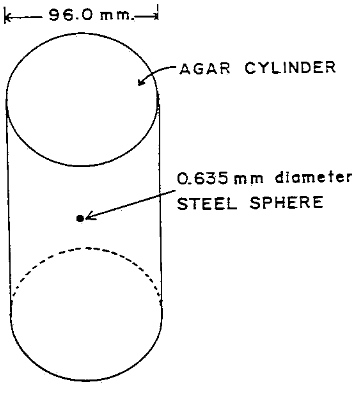

Fig. 2 Diagram of the agar target cylinder showing the position of the target sphere.

great distance (greater than 60 cm) from the ball bearing (table I). With such bursts, it was possible to isolate the signal corresponding to the scatter from the ball bearing. Also, variation of the pulse repetition rate allowed tests to verify that overlaps did not occur involving different bursts or spurious reflections from the tank walls.

To ensure that the experimental situation could be described theoretically by an asymptotic expression derived by Faran (Eq. (31) of [1]), the receiving transducer was placed at a great distance (60 cm) from the ball bearing. The receiving transducer was constrained to move in a circle which was centered about the ball bearing. Because the output signals of the receiving transducer were very small and contained high frequency interference from nearby electronic equipment, the signals were amplified and low pass filtered prior to making measurements. The cutoff edge of the filter was chosen to lie just above the frequency of the interrogating beam. The experimental data consisted of peak values of the amplified and lowpassed versions of the signals output by the receiving transducer when it was positioned at angles varying from about 170° to about 40° with respect to the incident beam. The angular increment between adjacent measurement positions was 3.6°.

The peak values of the amplitude waveform from the receiving transducers were normalized to similar peak values obtained from measuring the incident ultrasonic beam with the receiving transducer at the position of the scattering target and facing the transmiting transducer. The normalized values of the scattered pressure signals were expressed in terms of intensity and the Differential Scattering Cross Section (DSC) was calculated. The DSC is defined as:

$$\mathrm{DSC} = \left[ \frac{P_s(f,\theta)\ r}{Po(f)} \right]^2 \tag{1}$$

where f is the interrogating frequency, $\theta$ is the scattering angle, $P_s(f,\theta)$ and $P_o(f)$ are the scattered and incident acoustic pressure wave amplitudes respectively and r is the target-to-receiver distances (NOTE: By measuring the incident and scattered pressure waves with the same receiving transducers, any force-to-voltage transfer properties are removed from influencing the DSC).

The theoretical results shown in this study were obtained by programming Faran's expressions (Eq. (31) of [1]) for evaluation on a PDP 11/34 minicomputer (Digital Equipment Corporation, Maynard, MA 01754). Generation of this code was greatly facilitated through the use of mathematical recursive relationships for the Bessel, Neumann and Hankel functions provided in [5].

## III. RESULTS

Figures 3a,b,c,d show the experimental (triangles) and theoretical results (dark line) for the angular distribution of the DSC for interrogating frequencies of 1.0, 2.25, 3.5 and 5.0 MHz. The error bars on the experimental data represent the standard error of the mean DSC. Because the ratio of mean DSC to standard error showed little variation as a function angle, only a few representative error bars were included in the figures. As can be seen from the figures, there is excellent agreement between theory and experiment. The agreement is good not only in terms of the angular distribution, but also in terms of the absolute magnitude. The experimental results for scattering angles greater than 170° are absent due

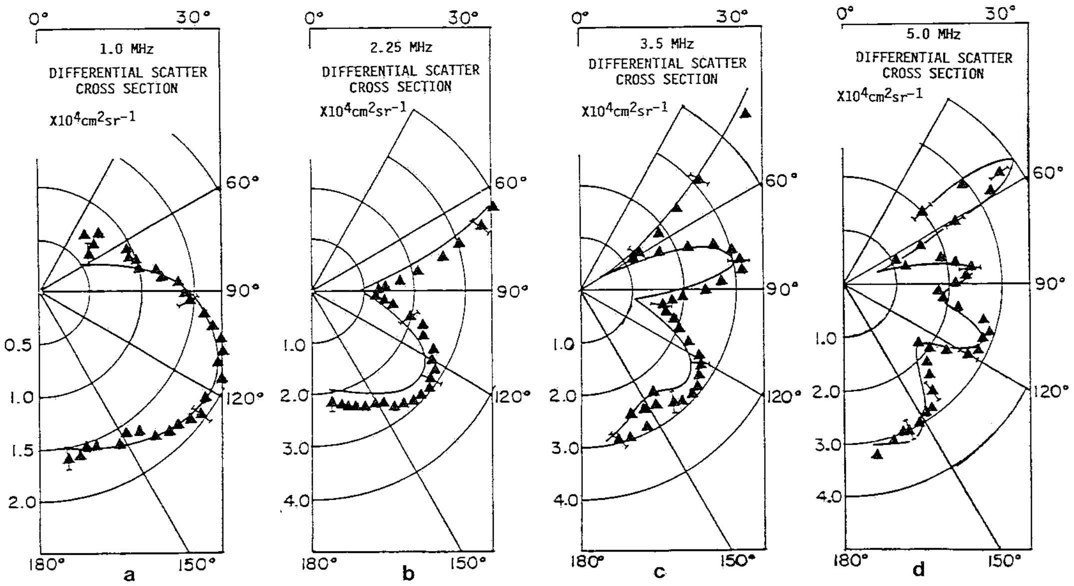

Fig. 3 Quantitative comparison between theory (——) and experiment (▲) for the DSC for a small ball bearing (diameter = 0.635 mm) embedded in agar: a) 1.0 MHz; b) 2.25 MHz; c) 3.5 MHz; d) 5.0 MHz.

to the inability to position the receiving transducer near the source transducer. Likewise, the results for scattering angles less than 44° are absent due to the interference caused by the incident beam.

## IV. DISCUSSION

Referring to figures 3a,b,c, and d, we see that the experimental results are adequately described by the theoretical expressions of Faran. This agreement was achieved even though the expressions from Faran's work was derived for a nonviscous media. The small, steel sphere-in-agar phantom was initially designed to be a simple and inexpensive laboratory tool to verify if an angle scattering apparatus was operating properly [4]. This type of phantom and experiment is easy to use because: 1) plane incident waves must be maintained only over the dimension of the target sphere; and 2) the results are independent of target to source or receiving transducer distance (as long as the results are expressed in terms of a DSC). In a subsequent experiment, Faran's expression was found to accurately describe the measured acoustic scattering behavior (at the same frequencies) for spherical targets ranging in size from tens of microns to 3 mm in diameter for material ranging from glass to graphite [4]. Theoretical knowledge of the single particle scatter behavior can and has been used to predict the angular distribution of scattered sound for graphite-loaded gelatin based phantom [4]. Thus, by knowing the properties of a population of discrete spherical scatterers embedded in a gelatin based ultrasound phantom, Faran's expression can be used to predict the expected scattering behavior [4].

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