Nonlinearity parameter estimation based on quantifying excess ultrasonic attenuation

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***Abstract*— The attenuation coefficient has potential in tissue characterization. Previously, it was observed that the accuracy of estimates of the attenuation coefficient slope (ACS) obtained using the spectral-log-difference technique (SLD) degraded due to nonlinear distortion. Based on these observations regarding estimates of the ACS, (i.e., the difference between ACS estimated at high and low pressures, which is called excess ACS in this work), a heuristic approach was devised for determining the nonlinearity parameter, B/A, of a medium.**

A total of 16 numerical phantoms using the k-Wave simulation package were constructed having spatially random distributions of acoustic density in a 3D grid to generate backscattered signals. To estimate ACS values from the phantoms, we used the SLD technique. The uniform numerical phantoms varied in their properties with B/A values of 6, 8, 10 or 12 and ACS values of 0.3, 0.7, 1.1 or 1.5 dB/cm/MHz. A focused source (f/2) with 1" focal length transmitted a 3.5-MHz centered Gaussian pulse at two different source pressures: 100 kPa (quasi-linear) and 1.3 MPa (nonlinear). Using one of the 16 phantoms as a sample ('sam') and another one as a reference ('ref'), we obtained 240 separate ACS estimates using the SLD for both the low and high pressures. The excess attenuation estimated was related to values of the Gol'dberg number, Γ, which is a parameter that predicts the degree of nonlinear distortion in plane waves.

The excess ACS versus $\log_{10}(\Gamma_{\text{sam}}/\Gamma_{\text{ref}})$ had a correlation coefficient of 0.96. The results indicate that a larger mismatch of Gol'dberg numbers between sample and reference resulted in larger excess ACS estimate. Therefore, whenever a large excess attenuation was observed, the Γ mismatch was also large. Using each of the 14 residual phantoms as a second reference ('ref2') with assumed known B/A and ACS leads to (ExcessAtt$_{\text{sam}}$)/(ExcessAtt$_{\text{ref2}}$) $\approx \log_{10}(\Gamma_{\text{sam}}/\Gamma_{\text{ref}})/\log_{10}(\Gamma_{\text{ref2}}/\Gamma_{\text{ref}})$ from which up to 3360 B/A$_{\text{sam}}$ could be estimated. Errors for estimated B/A were small ($\leq$10%) and high ($\geq$50%) for 31% and 22% of 1536 cases when the phantoms ('sam','ref', and 'ref2') had different preset ACS values. When the ACS was identical among the phantoms, the errors for the estimated B/A were relatively small ($\leq$10%) and high ($\geq$50%) for 46% and 6% of 96 cases, which can be explained by having better correlation.

***Index Terms*— Nonlinearity parameter, excess attenuation coefficient, quantitative ultrasound**

I. Introduction

The attenuation coefficient is a quantitative ultrasonic parameter that has potential to classify tissue or tissue state and is required to estimate other acoustic parameters like the backscatter coefficient. Particularly, attenuation estimation methods in pulse-echo mode include spectral-based approaches, i.e., in the frequency domain, because of the power-law form that of the attenuation coefficient versus frequency [1, p. 74]. Moreover, it has been observed in [2] and [3] that estimates of quantitative ultrasound parameters degrade when using spectral-based approaches in nonlinear acoustic regime, i.e., when large excitation pressure levels, but within the range of diagnostic imaging, are used in tissue. All tissues are nonlinear and that nonlinearity can be characterized by the nonlinearity parameter *B/A*. In [3] it was observed that inaccuracies in the attenuation coefficient slope estimates, called excess attenuation, maintained a strong linear correlation with the logarithm of the Gol'dberg ratio. The Gol'dberg ratio is the ratio of plane wave Gol'dberg numbers [4] of the unknown sample and reference phantom used for correction of beam diffraction. In this study we aim to verify if such correlation still holds for a larger set of 16 numerical phantoms with broader ranges of attenuation coefficient and nonlinearity parameter *B/A* (while in [3] only excess attenuation was estimated for 3 phantoms).

Furthermore, a heuristic estimation method for estimation of the nonlinearity parameter of unknown samples is presented based on the excess attenuation values. Hence, this provides a potential method for estimating *B/A* in pulse-echo mode that can be used potentially as a complementary metric in tissue characterization *in vivo*.

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II. Methods

A. Spectral log difference (SLD)

The attenuation coefficient vs frequency $\alpha_S(f)$ from a insonified uniform region of interest (ROI) of a sample can be obtained using power spectra from two windows of the ROI located at different depths z_p and z_d, proximal and distal, respectively, with $z_d > z_p$, as [5]

$$\alpha_S(f) = \alpha_R(f) + \frac{1}{4(z_d - z_p)} \log\left[\frac{S(f,z_p)S_R(f,z_d)}{S(f,z_d)S_R(f,z_p)}\right], \quad (1)$$

where $S(f,z_p)$ and $S(f,z_d)$ are the power spectra at depths z_p and z_d, respectively . The terms $\alpha_R(f)$, $S(f,z_p)$ and $S(f,z_d)$ are the attenuation coefficient of a well characterized reference phantom and power spectra from windows located at the same depths as in the sample. The reference phantom is used to compensate for beam diffraction effects of the source (e.g., focusing) and transducer impulse response.

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The SLD method assumes that acoustic nonlinearities are negligible, therefore, if a pressure level at the source was scaled no change should be observed in the backscattered RF data other than scaling. The attenuation coefficient slope (ACS) is the slope of a linear least-squared fit of $\alpha_S(f)$ estimated in (1) for the analysis frequency range, typically around the nominal frequency of the transducer.

B. Numerical simulations of RF data

RF data were obtained using the k-Wave toolbox [6]. Phantom media were simulated with 3D grids of 236×236×412 elements (20.3×20.3×35.5 mm) with uniform *B/A* and attenuation coefficient values but spatially random distributions of acoustic density (Gaussian distribution with mean 1000 Kg/m^3 and 2% standard deviation) to generate backscattered signals. Four B/A values of 6, 8, 10 or 12 and four ACS values of 0.3, 0.7, 1.1 or 1.5 dB/cm/MHz were used to generate a total 16 numerical phantoms. A focused transducer (f/2) with 1” focal length was configured in the 3D grid to transmit a 3.5 MHz short Gaussian pulse with 50% fractional bandwidth (–6dB). 2nd harmonic distortion was expected to be well described because 2nd harmonic (around 7MHz) was under the 8.7 MHz limit set by the simulation grid size. 100 RF lines (for spectral averaging) for each phantom were simulated at two different source pressure levels: 100 kPa and 1.3 MPa. Figure 1 shows the B-modes after filtering out the 2nd harmonic (to mimic the bandpass nature of a transducer). Note that no noticeable differences can be observed in the B-mode images versus source level in general when filtering out the harmonics.

C. Excess attenuation and B/A estimation

When increasing pressure levels, distortions in the frequency domain did occur and were captured through the variations in the attenuation coefficient slope estimated with the SLD method.

1) Excess attenuation: Comparison of attenuation coefficient slope estimates of an unknown sample from acquisitions at pressure levels 100 kPa and 1.3 MPa was computed as

$$\Delta_{\text{sample}} = \text{ACS}_{\text{sample,1.3MPa}} - \text{ACS}_{\text{sample,100kPa}}. \tag{2}$$

Given that 16 phantoms were simulated, we would choose one as a sample and another as a reference. Hence, up to 240 distinct pairs were obtained. For each pair, the excess attenuation was estimated to verify the linear correlation between excess attenuation and logarithm of Gol’dberg ratio.

2) B/A estimation: The Gol’dberg number for monofrequency plane waves is by definition [4]

$$\Gamma = \frac{k\beta M}{\alpha} = \frac{\omega_0 P}{\rho_0 c_0^3}\left(\frac{1+0.5\,\frac{B}{A}}{\alpha}\right), \tag{3}$$

where k is the wave number, β=1+0.5*B/A*, M is the Mach number, α is the attenuation coefficient, *P* is the source pressure, ρ_0 and c_0 are the equilibrium density and sound speed, and ω_0 is the angular frequency. Therefore the Gol’dberg ratio would be

$$\frac{\Gamma_{\text{sample}}}{\Gamma_{\text{reference}}} = \frac{1+\frac{1}{2}\frac{B}{A}_{\text{sample}}}{1+\frac{1}{2}\frac{B}{A}_{\text{reference}}}\frac{\text{ACS}_{\text{reference}}}{\text{ACS}_{\text{sample}}} \tag{4}$$

Then, due to the expected quasi-linear relation between the excess attenuation and the logarithm of Gol’dberg ratio, we approximated

$$\frac{\Delta_{\text{sample}}}{\Delta_{\text{reference}_2}} \approx \frac{\log_{10}\left(\frac{\Gamma_{\text{sample}}}{\Gamma_{\text{reference}}}\right)}{\log_{10}\left(\frac{\Gamma_{\text{reference}_2}}{\Gamma_{\text{reference}}}\right)}, \tag{5}$$

where the ‘reference$_2$’ corresponds to a second reference phantom with known acoustic properties. Then,

$$\Gamma_{\text{sample}} \approx \Gamma_{\text{reference}}\left(\frac{\Gamma_{\text{reference}_2}}{\Gamma_{\text{reference}}}\right)^{\frac{\Delta_{\text{sample}}}{\Delta_{\text{reference}_2}}}, \tag{6}$$

which can be rewritten as

$$1+\frac{1}{2}\frac{B}{A}_{\text{sample}} \approx \left(1+\frac{1}{2}\frac{B}{A}_{\text{reference}}\right)\left(\frac{\text{ACS}_{\text{sample}}}{\text{ACS}_{\text{reference}}}\right) \left(\frac{1+\frac{1}{2}\frac{B}{A}_{\text{reference}_2}}{1+\frac{1}{2}\frac{B}{A}_{\text{reference}}}\frac{\text{ACS}_{\text{reference}}}{\text{ACS}_{\text{reference}_2}}\right)^{\frac{\Delta_{\text{sample}}}{\Delta_{\text{reference}_2}}}, \tag{7}$$

from which the *B/A* of the unknown sample can be computed using true values of the phantoms (set in k-Wave or obtained with through-transmission methods and small signals in experiments). Using any of the 14 remaining phantoms as ‘reference 2’ we can have up to 2260 groups of three distinct phantoms (sample, reference, reference$_2$) whose RF data can be used in (7). Finally the fractional error (% error) of the *B/A* estimated with (7) with respect to the *B/A* true value set in k-Wave was be computed as

$$\%\ \text{error} = \left|\frac{\frac{B}{A}_{\text{sample, estimated}} - \frac{B}{A}_{\text{sample,true}}}{\frac{B}{A}_{\text{sample,true}}}\right| \times 100\% \tag{8}$$

III. Results

Figure 2 shows the excess attenuation estimated by using a source peak pressure 1.3 MPa instead of 100 kPa in nonlinear media. For 240 pairs of sample and reference the excess attenuation is highly correlated (*r*=0.96) with the logarithm of Gol’dberg ratio between sample and reference. The largest deviation was 1.39 dB/cm/MHz for the case when the Gol’dberg ratio was 8.7. Symmetry is observed due to duplication of the phantom pairs (i.e., reference swapped with sample). The minimum deviation was –1.39 dB/cm/MHz for the case when the Gol’dberg ratio was 0.11. The results indicate that a larger mismatch of Gol’dberg numbers between sample and reference resulted in larger

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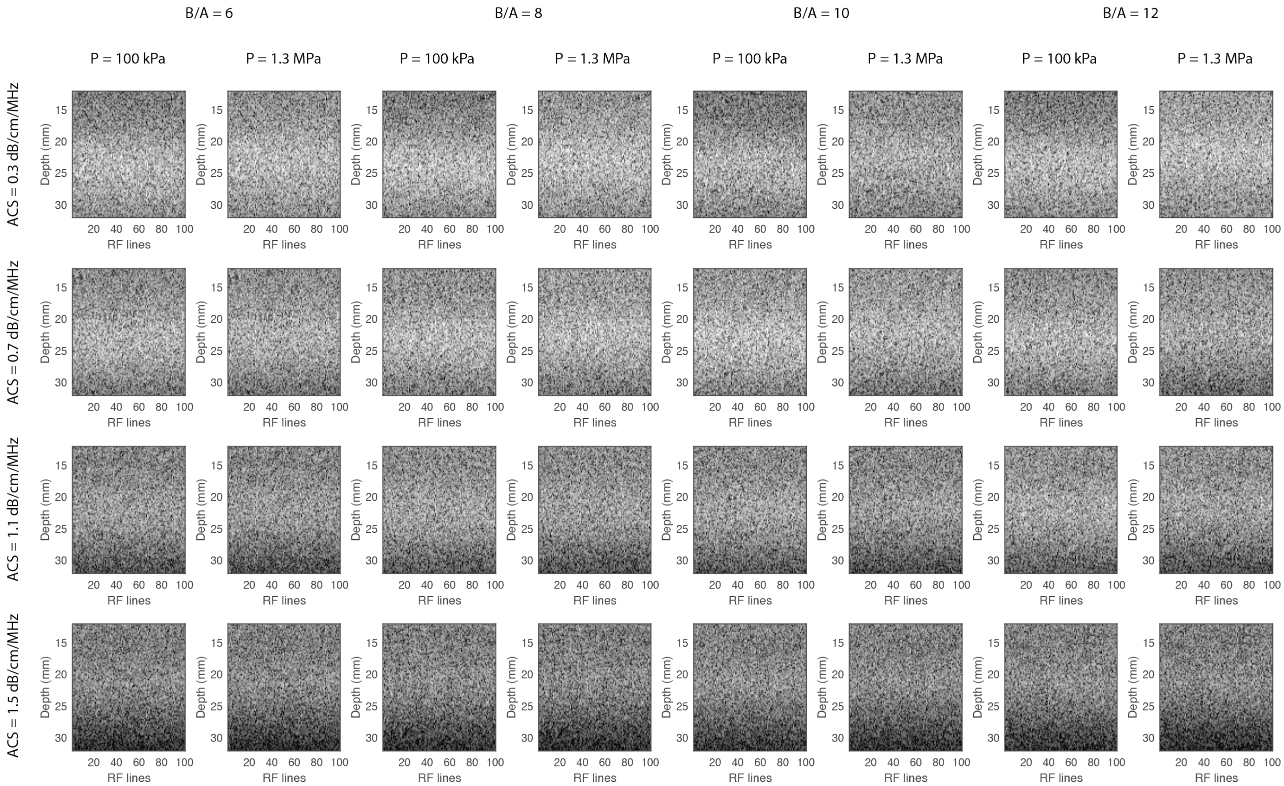

Fig. 1: B-mode images of the computer simulated RF lines at two different pressure levels: 100 kPa and 1.3 MPa (normalization with respect to the respective maxima is performed). Bottom row show the case of the highest attenuation phantoms (ACS = 1.5 dB/cm/MHz) with stronger shadowed regions at larger depths. Visual changes between different pressure levels appear to be generally unnoticeable except in the case with smaller attenuation ACS=0.3 dB/cm/MHz and the largest nonlinearity parameter B/A = 12 (top right phantom) which is the phantom expected to have the largest Gol'dberg number.

excess ACS estimate. Therefore, whenever a large excess attenuation was observed, the Γ mismatch was also large.

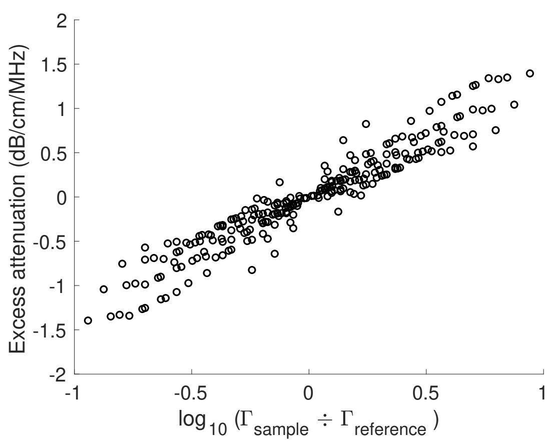

Fig. 2: Excess attenuation estimated by using a source peak pressure 1.3 MPa instead of 100 kPa in nonlinear media. For 240 pairs of sample and reference the excess attenuation is highly correlated (r=0.92) with the logarithm of Gol'dberg numbers between sample and reference.

The % error of estimated B/A were small ($\leq$10%) and large ($\geq$50%) for 31% and 22% of 1536 cases when the phantoms (sample, reference, and reference$_2$) had different preset ACS values, as can be observed in the left boxplot of Fig. 3. However, when the ACS was the same between the phantoms and only had mismatches coming from *B/A*, the errors estimated were relatively small ($\leq$10%) and large ($\geq$50%) for 46% and 6% of 96 cases, as shown in the right boxplot of Fig. 3. Figure 4 corresponds to excess attenuation measured with pairs, sample-reference, that shared the same ACS and it was observed that excess attenuation was linearly correlated with the logarithm of Gol'dberg ratio. However, the assumption of linearity used in (5) held stronger when the ACS was shared between the sample and the reference, which might explain why the errors on the right boxplot of Fig. 3 are lower.

IV. Discussion

This work presents a *B/A* estimation approach that uses the backscattered signals acquired in pulse-echo ultrasound and a common method for attenuation estimation (SLD). The method assumed that excess attenuation of ACS values at different pressure levels: low (quasi-linear) and high (nonlinear)

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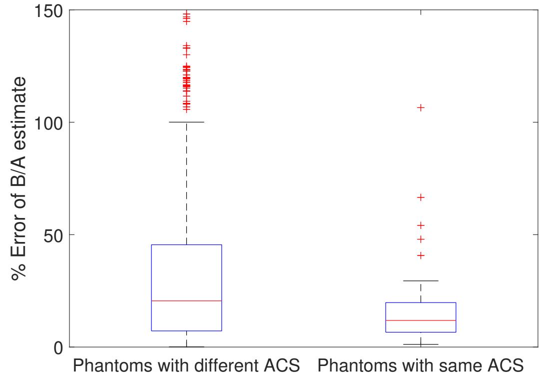

Fig. 3: Boxplot of the estimated fractional error of the *B/A* estimates. The left boxplot shows the error when phantoms involved in the estimation of Eq. 6 are different whereas the right boxplot shows the case when phantoms only had distinct nonlinearity parameters. Errors were potentially larger when the phantoms did not share the same ACS.

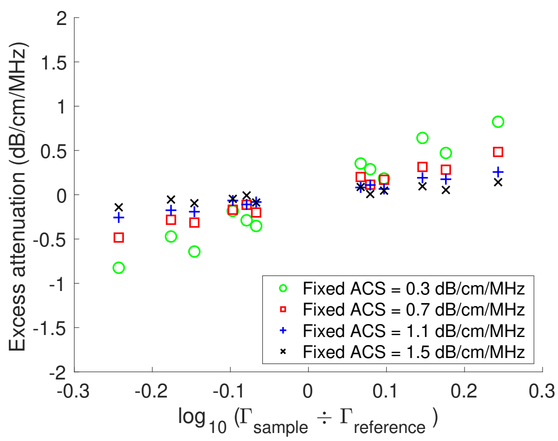

Fig. 4: Excess attenuation estimated by using a source peak pressure 1.3 MPa instead of 100 kPa in nonlinear media. Pairs, sample-reference, shared the ACS (either one of the four values available) and only had different *B/A*. For the subsets with the same ACS, the correlation was larger than 0.98, i.e, better than in Fig. 2 except for ACS=1.5 dB/cm/MHz with r=0.93.

was caused by the acoustic nonlinear distortion. This method used available RF data to compute excess attenuation using two excitation pressure levels (one low and another high) of an already existing ultrasonic system.

However, it was observed that to improve the accuracy of this method the phantoms involved in the estimation should have the same attenuation coefficients as the sample. Although for an *in vivo* application, the phantoms could have sound speed very close to that of soft tissues, the attenuation coefficient can be much different than the target tissue. Hence, this method requires the construction of up to two ad-hoc phantoms with similar attenuating properties as the sample tissue and with *B/A* characterized by a through-transmission method. Furthermore, even if the attenuation coefficient was exactly the same, only for about half of these cases the error of *B/A* estimated was below 10%. Further analysis is required to compensate for the non-perefect linear trend evidenced in Fig. 4 and for the largest ACS where excess attenuation tend to zero and might be more sensible to errors.

In conclusion, the findings suggest that excess attenuation could be used for estimation of *B/A* in pulse-echo ultrasound providing that attenuation of media involved have similar attenuation coefficients. Further research is required to analyze fractional errors when mismatches are not zero but very small and to compensate for non-perfect linear relation between excess attenuation and Gol'dberg ratio.

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