Application of l_1 Regularization for High-Quality Reconstruction of Ultrasound Tomography

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Abstract — **Ultrasound tomography based on inverse scattering has the capability to resolve structures which are smaller than the wavelength of the incident wave, as opposed to conventional ultrasound imaging using echo method. Some material properties such as sound contrast are very useful to detect small objects. Born Iterative Method (BIM) based on first-order Born approximation has been introduced as an efficient diffraction tomography approach. However, this method has a high complexity because it has to solve large iterative forward and inverse problems. In this paper, we propose to replace Tikhonov regularization by l_1-regularized least squares problem (LSP) in solving the inverse problem in BIM. As a result, the quality of reconstruction is improved and the complexity is reduced.**

Keywords — **Ultrasound, tomography, inverse scattering, Born Iterative Method, Tikhonov regularization.**

I. INTRODUCTION AND STATE-OF-THE-ARTS

Ultrasound imaging and tomography play important roles in clinical detection. Ultrasound image acquisition is mostly based on a pulse echo method that uses the time of light energy reflected by the boundaries of the object under imaging [2]. By extending the number of angles around the object, inverse scattering based techniques offer better quality of image reconstruction under strong scattering [2].

Works in ultrasound tomography often focus on calculating the size of tissue (scattering area) and the speed of sound crossing the object being imaged. At present, there are only a few commercialized tomography devices. The reason is that state-of-the-art inverse scattering techniques have high computational complexity as well as limited efficiency. Born Iterative Method (BIM) and Distorted Born Iterative Method (DBIM) are well-known for diffraction tomography [1].

DBIM is more sensitive to noise, though it offers faster convergence as compared to that of BIM. In addition, the computational complexity of these methods is high due to their use of iterative forward and inverse processes. In [2], edge detection during the iterative process was introduced in order to speed up the convergence and to enhance the quality of reconstruction, but the complexity and noise sensitivity issues remain. In [3], the multi-level fast multi-pole algorithm (MLFMA) was applied to the forward solver for further speed up the reconstruction process. However, MLFMA requires high set-up costs that make it difficult to implement in practice.

In conventional BIM, Tikhonov regularization (a.k.a., l_2 regularization) [4] was employed in solving the inverse problem. This method does not de-noise well, as we will later show in this paper. Based on this observation, we realize that if we solve the inverse problem well in terms of the speed and the quality of reconstruction, then we can reduce the number of the forward problem that follows in the iterative process. Thus, in this paper, we replace Tikhonov regularization by the l_1-regularized least squares problem (LSP) [5]. We also show that this modification helps improve the quality of reconstruction and reduce the computational complexity.

II. MATERIALS AND METHODS

A. Born Iterative Method (BIM)

Fig. 1 schematically shows geometrical and acoustical configuration of the ultrasound tomography system.

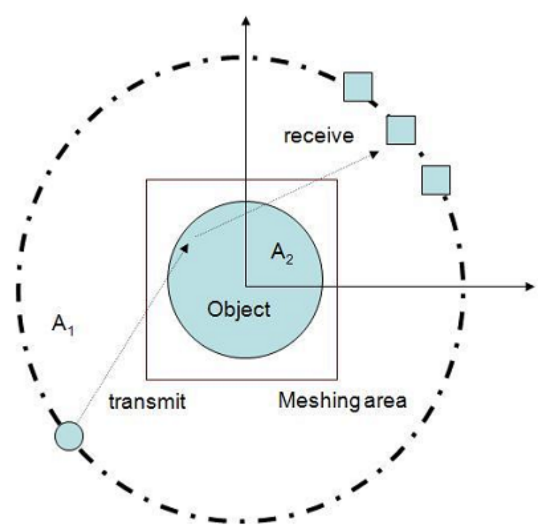

Fig. 1 Geometrical and acoustical configuration.

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The region of interest (ROI) consists of the reconstructing object, centered at the origin of a 2-D space and discretized into $N \times N$ square pixels of side . The number of transmitters and receivers are N_t and N_r , respectively. With the circular scatter area as shown in Fig. 1, the object function can be computed by

$$O(r) = \begin{cases} \omega^2\left(\frac{1}{c_1^2} - \frac{1}{c_0^2}\right), |r| \le R \\ 0, \quad |r| > R \end{cases} \tag{1}$$

Where c_1 and c_2 are the speed of sound in the object and in the water, respectively, f is the ultrasound frequency, ω is the angular frequency (i.e., $\omega = 2\pi f$), and R is the radius of the object.

We set up a measurement configuration for transmitters and receivers in order to obtain the scattered data. At an instance, only one transmitter and one receiver are active to for a corresponding measured data value. Assumed that the density variation is negligible, the inhomogeneous wave equation is given by

$$\left(\nabla^2 + k_0^2(r)\right)p(r) = -O(r)p(r), \tag{2}$$

where $k_0 = \omega / c_0$ is the wavenumber in the reference medium (i.e., water), and $p(r)$ is the total pressure field.

By solving (2), the scattering pressure can be obtained in intergral form, in term of the Green's function, as

$$p^{sc}(r) = p(r) - p^{inc}(r) = \iint O(r)\, p(r) G(|r - r'|) \tag{3}$$

where $p^{inc}(r)$ in the incident pressure, and is the free space Green's function. Equation (3) can be solved by method of moment utilizing the sinc basis and delta functions [6]. The pressure ingrid points can be presented in an $N^2 \times 1$ vector

$$\boldsymbol{p} = \boldsymbol{p}^{inc} + \boldsymbol{C}\boldsymbol{D}(\overline{\boldsymbol{O}})\overline{\boldsymbol{p}}, \tag{4}$$

and the scattered pressure can also be calculated in a scalar value

$$\boldsymbol{p}^{sc} = \overline{\boldsymbol{B}}_i \boldsymbol{D}(\overline{\boldsymbol{O}})\overline{\boldsymbol{p}} \tag{5}$$

where $\overline{\boldsymbol{B}}_i$ is a $1 \times N^2$ vector converted from a matrix formed by Green's coefficient $G_0(r, r')$ from each pixel to the i^{th} receiver, $\boldsymbol{C}$ is an $N^2 \times N^2$ matrix formed by Green's coefficient among all pixels in the meshing area, and $\boldsymbol{D}(\cdot)$ is an operator converting a vector into a diagonal matrix. Detailed calculations of $\overline{\boldsymbol{B}}_i$ and $\boldsymbol{C}$ can be found in [6].

If N_t transmitters and N_r receivers are used, then the scattered pressure signal vector of size $N_t N_r \times 1$ can be obtained from (5) as

$$\boldsymbol{p}^{sc} = \overline{\boldsymbol{B}}_i \boldsymbol{D}(\overline{\boldsymbol{O}})\overline{\boldsymbol{p}} = \boldsymbol{M}\overline{\boldsymbol{O}} \tag{6}$$

where $\boldsymbol{M} = \boldsymbol{B}\boldsymbol{D}(\overline{\boldsymbol{p}})$ is the matrix whose size is $N_t N_r \times N^2$.

These data are processed using BIM to reconstruct the speed of sound contrast. By this way, we can detect if any tissue exists in the medium. BIM uses Born approximation to compute iterative solutions of a nonlinear inverse scattering problem. If we use multiple transmitters and receivers, the object function $\overline{\boldsymbol{O}}$ can be estimated using Tikhonov regularization

$$\overline{\boldsymbol{O}} = arg\,\min_{\overline{O}} \|\overline{\boldsymbol{p}} - \overline{\boldsymbol{M}}\overline{\boldsymbol{O}}\|_2^2 + \gamma\|\overline{\boldsymbol{O}}\|_2^2 \tag{7}$$

where γ is the regularization parameter that needs to be chosen carefully because it contributes mostly to the stability of the system [7]. High values of γ make the reconstructed image rough. However, small values of γ lead to high computational complexity.

The solution of Tikhonov regularization problem can be calculated directly, or indirectly using an iterative method. The iterative method is more efficient than the direct one, especially when $\overline{\boldsymbol{M}}$ is sparse or has a special form (e.g., partial Fourier or wavelet matrices).

B. Proposed Method

Recently, l_1-regularization based methods have been applied widely for sparse signal reconstruction or feature selection. In [5], authors extended to l_1-regularized least squares programs (LSP) in order to reformulate them as convex quadratic programs, and then solved them by a specialized interior-point method. The advantage of this approach is that it can solve both sparse and dense problems with high accuracy at some small additional computational costs.

The object function $O(r)$ contains the information of the sound contrast of the target image. In Section II.A, $O(r)$ was updated at each iteration by Tikhonov regularization. Now, we reformulate the problem as the following LSP in order to improve the quality of solution $O(r)$:

$$\overline{\boldsymbol{O}} = arg\,\min_{\overline{O}} \|\overline{\boldsymbol{p}} - \overline{\boldsymbol{M}}\overline{\boldsymbol{O}}\|_2^2 + \gamma\|\overline{\boldsymbol{O}}\|_1 \tag{8}$$

where $\overline{\boldsymbol{M}}$, $\overline{\boldsymbol{p}}$, and γ are respectively the measurement matrix, the scattered field vector, and the regularization parameter. Our method can be summarized in Algorithm 1. In this algorithm, the relative residual error is defined by

Algorithm 1. l_1-Regularization for high-quality reconstruction.

Choose initial values: $\bar{\boldsymbol{O}}_0 = \bar{\boldsymbol{0}}$ and $\bar{\boldsymbol{p}}_0 = \bar{\boldsymbol{p}}^{inc}$ as shown in (10)
Calculate $\bar{\boldsymbol{B}}$ and $\bar{\boldsymbol{C}}$ [6]
while $n < N_{\max}$ or RRE $< \epsilon$
1. Calculate $\bar{\boldsymbol{p}}$ corresponds to $\bar{\boldsymbol{O}}_n$ using (4)
2. Calculate RRE corresponds to $\bar{\boldsymbol{O}}_n$
3. Calculate a new value of $\bar{\boldsymbol{O}}_{n+1}$ by solving (8)
4. $n = n + 1$

End

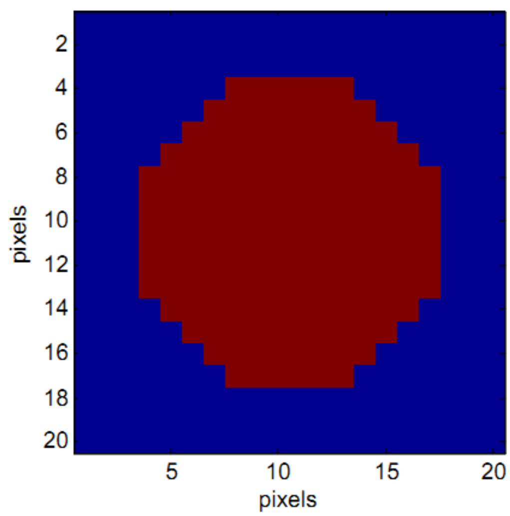

Fig. 2 Ideal object function.

$$RRE = \frac{\|\bar{\boldsymbol{M}}\bar{\boldsymbol{O}} - \bar{\boldsymbol{p}}^{sc}_{measured}\|}{\|\bar{\boldsymbol{p}}^{sc}_{measured}\|} \tag{9}$$

RRE is computed in each iterative step.

The loop will be terminated when RRE is smaller than a desired tolerance or when the number of iterative steps reaches some preset N_{max}.

III. Results and Discussions

Simulated data were created for infinite circular cylinderusing 20×20 pixels. The number of transmitters and receivers are 40 and 20, respectively. The scattering field is affected by 5% Gaussian noise. The radius of the cylinder is 5λ, the frequency of ultrasound signal is 1 MHz, and the sound contrast is 2%.

The incident pressure for a Bessel beam of zero order in two dimensional case is

$$\bar{\boldsymbol{p}}^{inc} = J_0(k_0|\boldsymbol{r} - \boldsymbol{r}_k|) \tag{10}$$

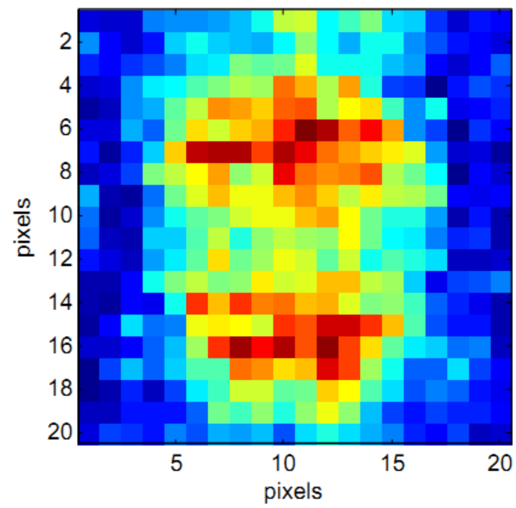

Fig. 3 Reconstructed object function after 5 iterations, using conventional method.

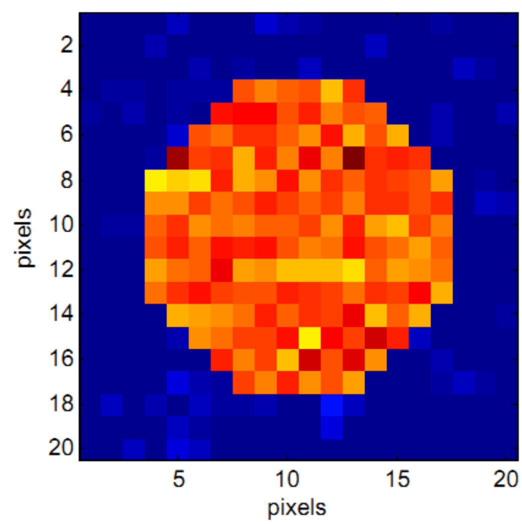

Fig. 4 Reconstructed object function after 5 iterations, using proposed method.

where J_0 is the 0[th] order Bessel function and $|\boldsymbol{r} - \boldsymbol{r}_k|$ is the distance between the transmitter and the k^{th} point in the ROI.

Fig. 2 is the ideal object function corresponding to (1). Figs. 3 and 4 are reconstructed object functions after 5 iterations, using the conventional and proposed methods, respectively. They indicate that using l_1-regularization can greatly enhance the quality of reconstruction as compared to using the l_2-regularization.

We can also compare the performance the conventional and proposed systems using the performance error

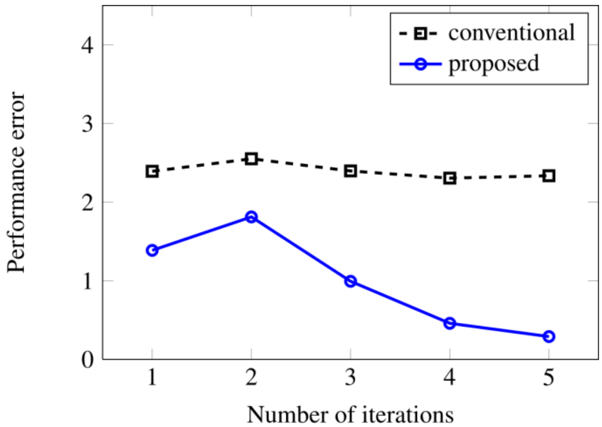

Fig. 5 Performance comparison between the conventional and proposed methods.

$$e = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\left|C_{ij}-\hat{C}_{ij}\right|}{C_{ij}} \quad (11)$$

where C_{ij} and $\hat{C}_{ij}$ are the ideal and estimated ultrasound velocities at pixel (i,j).

Fig. 5 presents the error performance of these two methods. It shows that, by applying the LSP, the reconstruction quality has been improved. Although the computational complexity for each LSP reconstruction is 1.2 times larger than that in the conventional one, we have reduced the overall computational complexity for the same reconstructed quality. In the conventional method, after the 4th BIM iteration, the error performance is saturated due to the noise background. It means that even if we further increase the number of BIM iterations, we could hardly improve the reconstruct quality.

IV. Conclusions

This paper has successfully applied LSP to BIM in order to reduce the computational complexity and to improve the quality of the construction of the sound contrast. The BIM is chosen here to solve the inverse scattering problem due to its strength against noise. A simulation scenario of sound contrast reconstruction has been performed to show the outperformance of this method. This work will be further developed using experimental data.

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