Observation of generalized Wolf shifts in short pulse spectroscopy

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Experimental observations of recently predicted correlation-dependent, propagation-induced changes in the spectrum and cross-spectra of a cyclostationary field are reported. An acoustic Young's experiment was conducted in which sources were driven by stationary signals modified by a square wave to make the process cyclostationary. The degree of correlation of the underlying stationary processes were controlled to change the spatial coherence of the sources. The far-zone generalized spectra changed with changing degree of correlation. © *2011 American Institute of Physics*. [doi:10.1063/1.3599842]

Interferometry and spectroscopy are rooted in the theory of stationary (at least in the wide sense) random processes. That is, the autocorrelation function of the field is assumed to be a function only of time delay and consequently the power spectrum is the Fourier transform of the autocorrelation.[1–3] Sources that generate fields only a few optical cycles in duration are certainly not well modeled in a stationary theory and must be considered in a general setting.[4] Just as a theory of stationary random processes provides a context to understand standard spectroscopy, a theory of nonstationary fields is needed to treat problems in short-pulse and comb spectroscopy.[5] However, trains of pulses such as appear in comb spectroscopy may yet exhibit discrete time-translational invariance, at least in a statistical sense, and so some of the more useful features of the stationary theory may be retained. To that end, recent work has brought the considerable body of results in the theory of cyclostationary random processes to bear on problems of propagation and interference in optics (Refs. 6 and 7) and spectroscopy.[8]

The power spectrum of the optical field generally changes upon propagation in a manner dependent on the spatial correlations of the source, an effect known as the Wolf shift,[9–12] with implications in spectroscopy and radiometry.[13] The effect also appears in scattering from random media, in certain cases generating Doppler-like shifts.[14] The original prediction of correlation-dependent changes to the spectrum of light for statistically stationary fields was verified in an acoustic experiment.[15]

In cyclostationary random processes, the power spectrum is replaced by a collection of generalized spectra (reviewed below). Changes in the generalized power spectra and the two-frequency cross-spectral density have been predicted for fields generated by periodically pulsed optical fields.[8] In this letter we report the observation of these spectral changes in an experiment analogous to the original observation of the Wolf shift for stationary fields.

Consider a stochastic, cyclostationary source density $\sigma(\mathbf{r},t)$ with mutual coherence function $\Gamma_\sigma(\mathbf{r}_1,\mathbf{r}_2,t-\tau,t)=\langle\sigma^*(\mathbf{r}_1,t-\tau)\sigma(\mathbf{r}_2,t)\rangle$, where the brackets denote an ensemble average. Cyclostationary processes exhibit a discrete time translation symmetry rather than the continuous symmetry of stationary processes, so $\Gamma_\sigma(\mathbf{r}_1,\mathbf{r}_2,t-\tau,t)=\Gamma_\sigma(\mathbf{r}_1,\mathbf{r}_2,t-\tau+T_0,t+T_0)$ for some fixed period T_0. The Fourier transform of the mutual coherence function is the two-frequency cross-spectral density, $W_\sigma(\mathbf{r}_1,\mathbf{r}_2,\omega_1,\omega_2)=\int dt_1 dt_2 \Gamma_\sigma(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)e^{i(\omega_2 t_2-\omega_1 t_1)}$. The two-frequency power spectral density of the source may be written

$$W_\sigma(\mathbf{r}_1,\mathbf{r}_2,\omega,\omega+\Omega)=\sum_n \tilde{C}_n(\mathbf{r}_1,\mathbf{r}_2,\omega)\delta(\Omega-n\omega_0), \quad (1)$$

where $\omega_0=2\pi/T_0$. Equation (1) is a generalization of the Wiener-Khintchine theorem, the regular form being obtained by taking the $T_0\to\infty$ ($\omega_0\to 0$) limit and then identifying that $\tilde{C}_0(\mathbf{r},\mathbf{r},\omega)$ is the usual power spectral density. The $\tilde{C}_n(\mathbf{r},\mathbf{r},\omega)$ are generalized spectral densities and the $\tilde{C}_n(\mathbf{r}_1,\mathbf{r}_2,\omega)$ are generalized cross-spectral densities. In the usual form of the Wiener-Khintchine theorem, the delta function $\delta(\Omega)$ indicates that the field is uncorrelated across frequencies. In this generalized form, it is clear that the field is correlated only at discretely-spaced frequencies.

In general, spectra cannot be determined at arbitrary points from the known spectra of the source or even the spectra on some plane. Instead, it is necessary to propagate the two-point, and for nonstationary processes, two-frequency, cross-spectral density. It has been shown that the two-point, two-frequency cross-spectral density satisfies the Wolf equations.[16,17] The cross-spectral density may be propagated by standard Green's function methods and the two-frequency spectral density in the far-zone of the source is given by the expression (Ref. 8)

$$S(\mathbf{r},\omega,\omega+\Omega)=\sum_n S_n(\mathbf{r},\omega)\delta(\Omega-n\omega_0), \quad (2)$$

where

$$S_n(\mathbf{r},\omega)=\frac{e^{i(k_{2,n}-k_1)r}}{r^2}\tilde{\mathcal{C}}_n(-k_1\hat{r},k_{2,n}\hat{r},\omega)$$

and $\mathbf{r}=r\hat{r}$, $k_1=\omega/c$, $k_{2,n}=(\omega+n\omega_0)/c$, and $\tilde{\mathcal{C}}_n$ is the six-dimensional spatial Fourier transform of $\tilde{C}_n$. The S_n may be measured by heterodyne methods at optical frequencies, or

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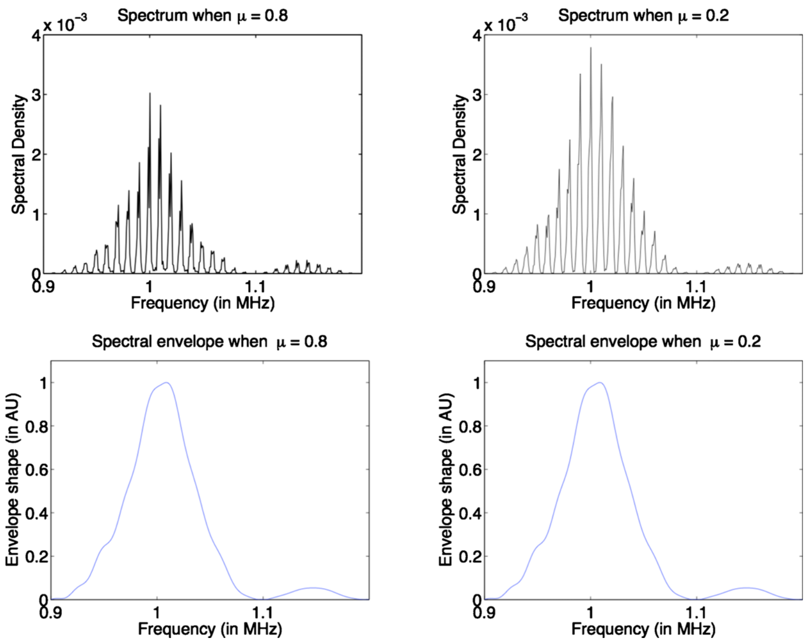

FIG. 1. (Color online) Plots of the spectral density, $S_0(\omega)$ (upper panels), and the envelope of the spectral density (lower panels) for the experiment where the degrees of coherence between the fields emitted by the two transducers are 0.8 (left panels) and 0.2 (right panels).

may be computed from direct measurements of the fields at low frequencies over long times. Each generalized spectral density may experience a different spectral shift. Thus $\widetilde{C}_0$ may undergo the standard Wolf shift, and a sequence of generalized Wolf shifts are manifest in each of these generalized spectra, S_n.

Generalized Wolf shifts have now been observed in an acoustic experiment and are reported in this letter. Two unfocused circular pistons with nominal center frequency of 1 MHz and reported diameter of 0.125 in., placed 2 cm apart, were used as sources for a cyclostationary acoustic field. The acoustic field was measured on a third transducer: a 1 mm wide by 1 cm high rectangular transducer with nominal center frequency of 5 MHz, 20 cm away from the sources in a large tank of water. The transducers were driven with intrinsically stationary (Refs. 7 and 18) signals. That is, two partially correlated, stationary signals were modulated by a square wave to produce partially correlated cyclostationary signals to drive the transducers. The spectral degree of correlation, μ_{12}, of the stationary signals was controlled using the method described in Ref. 19. The power spectral density of the stationary signals were, to a good approximation, Gaussian, centered at 1 MHz with a width of 8.33 kHz, corresponding to a correlation time of 120 μs. The stationary signals were modulated by a sequence of 100 square pulses with 10% duty cycle, and pulse-to-pulse period of 100 μs. The echo signal from the rectangular transducer was received by a pulser-receiver at 50 MHz. Measurements were made for two degrees of correlation, μ_{12}=0.8 and μ_{12}=0.2.

In Fig. 1, the spectral density and the envelope of the spectral density are plotted for μ_{12}=0.8 (on the left) and μ_{12}=0.2 (on the right). The two spectral densities differ in overall scale, but the envelopes are the same; both peak at 1.02 MHz. That is, the zeroth-order spectrum is not signifi-

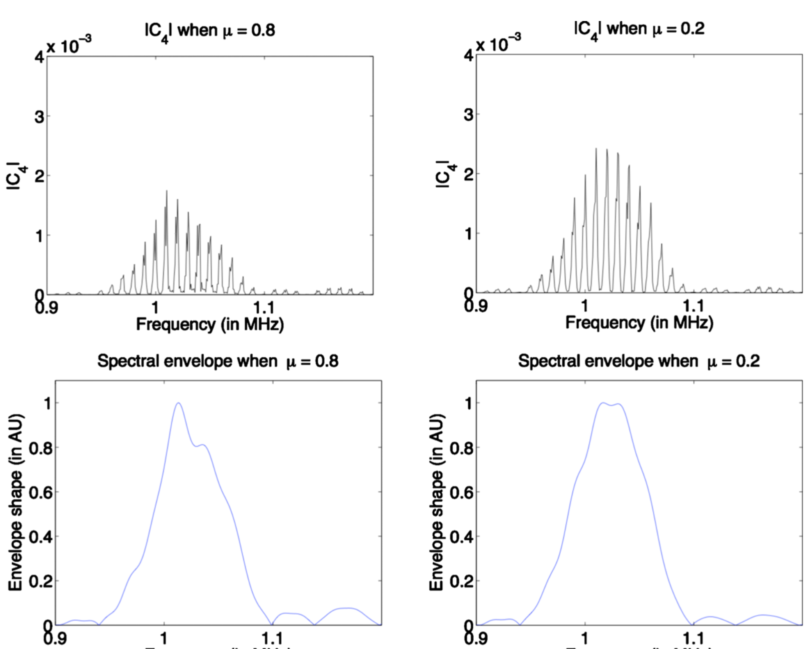

FIG. 2. (Color online) Plots of the fourth-order spectral density, $|S_4(\omega)|$ (upper panels), and the envelope of the cross-spectrum (lower panels) for the experiment where the degrees of coherence between the fields emitted by the two transducers are 0.8 (left panels) and 0.2 (right panels).

cantly shifted by changing the degree of correlation.

In Fig. 2, the magnitude of the fouth-order spectral density, $|S_4(\omega)|$, and the envelope of the spectrum are plotted for $\mu_{12}=0.8$ (on the left) and $\mu_{12}=0.2$ (on the right). In this case, the spectral envelopes are markedly different demonstrating the correlation-dependent generalized Wolf shift. For $\mu_{12}=0.8$, the $|S_4(\omega)|$ exhibits a large peak at 1.01 MHz and a smaller peak at 1.02 MHz. For $\mu_{12}=0.2$, $|S_4(\omega)|$ exhibits two peaks of nearly equal amplitude at 1.01 and 1.02 MHz.

The results presented here demonstrate the generalized Wolf shifts and two key aspects of them. First, the fouth-order spectral density shows a significant change dependent on the degree of correlation in the two sources. Second, in the same experiments, the zeroth-order spectral density is unchanged, demonstrating that the generalized spectral densities can undergo different correlation-induced spectral changes.

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