Low-frequency sound wave parameter measurement in gravels

George W. Swenson Jr. a,*, Michael J. White b, Michael L. Oelze a

a Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, 1308 West Main Street, Urbana, IL 61801, United States
b US Army Engineer Research and Development Center, Construction Engineering Research Laboratory, P.O. Box 9005, Champaign, IL 61826, United States

Abstract

Experiments were performed to measure the various sound-wave propagation parameters of two kinds of gravel, a crushed limestone and a stream-formed “pea gravel”, at frequencies between 25 and 200 Hz. The apparatus was a very large standing-wave tube with an array of microphones along its entire length. Measurements were made with the tube in the vertical position, filled to various depths, with a pressure-doubling barrier at the bottom. Values were determined for wavelength and attenuation in the gravel and for input impedance at the boundary between gravel and air. The basic analysis method was to fit the parameters of a presumed Green’s function representing the standing wave in the tube to the measured sound pressure amplitudes measured by the microphones. This worked well for some frequencies and some depths of gravel. In other cases, various modifications of the method gave more consistent results. For crushed limestone, the estimates of phase velocity and attenuation increased monotonically with frequency, ranging between 150 and 250 m/s and 0.1 and 0.8 Np/m, respectively. For pea gravel, the estimates increased monotonically with frequency, with values between 160 and 205 m/s and 0.4 and 1.1 Np/m, respectively.

1. Introduction

This document reports the experimental results of a study of low-frequency (25–200 Hz) sound-wave propagation in two representative types of “gravel”: a coarse crushed limestone and a smaller, stream-formed “pea gravel”. Such materials have been considered as media for the absorption of sound energy from military activities such as demolition and artillery training, and there is a paucity of available information on their sound absorption and propagation properties at the low frequencies characteristic of the noise of such activities. While theories exist for sound waves in porous media [1], very few have been tested for the prediction of propagation or absorption effects in the low-frequency range under consideration [2–4]. Thus there exists a need for basic experimental data that can be used for the prediction of sound absorption and propagation in various contexts.

Several experiments have reported acoustic characteristics of rigid, air-filled porous media. Papers reporting measurements of dispersion, attenuation and impedance for gravel, sand, or soils include Attenborough [5], Bolen and Bass [6], Dickinson and Doak [7], Embleton et al. [8], Frederickson et al. [3], Hess et al. [9], Hickey and Sabatier [10], Hutchinson-Howorth et al. [11], Song and Bolton [12], Oelze et al. [13], and Lu et al. [14]. Of those papers, only three report data for frequencies as low as 100 Hz, and only Bolen and Bass [6] report data for 40 Hz. The level-difference technique used by Hess et al. [9], and the propagation inverse-modeling approach of Bolen and Bass [6], Dickinson and Doak [7], and Embleton et al. [8] become insensitive to the value of ground impedance for low frequencies such as 40 Hz and 100 Hz. The impedance tube first used by Dickinson and Doak [7] is difficult to manage for outdoor measurements, and requires lengths comparable to one-quarter wavelength in air for the lowest frequency of interest.

Sound propagation experiments within and over porous media present many difficulties; for example, from the susceptibility of buried microphones to damage from the porous medium, from mathematical complications arising from three-dimensional geometry, and from the lack of structural rigidity of many porous materials. Therefore, an experimental procedure was developed to acquire acoustic wave propagation data at low frequency, which could avoid the above-mentioned difficulties.

2. Description of facilities

The US Army Engineer Research and Development Center, Construction Engineering Research Laboratory (ERDC/CERL) has a suite of acoustical impedance-measuring facilities intended for use at low frequencies [15,16]. One of the instruments is a Kundt’s tube (or “Impedance Tube”), consisting of a 7.3-m long, 76-cm diameter Schedule 80 steel pipe, with microphone ports every 20 cm along a straight line in the wall of the tube (see Figs. 1 and 2). There is 30 cm of separation between the loudspeaker face and the topmost...
microphone. The lowest microphone is 21 cm above the terminating plate. Schedule 80 pipe in this size has a wall thickness of 12.7 mm. A heavy steel lid is bolted to one end of the tube, presumably forming a rigid termination, and a 12-in loudspeaker (MTX Thunder T9512-04) in a heavy plywood baffle forms the driving source at the other end. Instantaneous sound pressures are measured by 32 precision microphones in the ports and voltages corresponding to the instantaneous sound pressures are recorded by two 16-channel oscilloscopes (Yokogawa DL750 ScopeCorder). The loudspeaker can be excited by continuous-wave signals at discrete frequencies or by broad-band random noise. A variety of laboratory-grade instrumentation microphones in one-inch and half-inch sizes (Bruel and Kjaer and Larson-Davis) were used in different combinations for different data-collecting runs. Before each run, all microphones were calibrated by a Brüel and Kjaer type 4228 pistonphone at 250 Hz. No differences have been observed in the performance of any of the microphones.

For experimental investigation of the properties of the porous medium, gravel in this case, the tube is filled part way, leaving a substantial depth of air above the gravel. It is assumed that the porous medium is homogeneous and isotropic on a length scale of a small fraction of a wavelength, and that the sound field can be modeled by the classical wave equation. The radial dimension of the tube is expected to restrict the sound field to plane-wave modes along the length of the pipe at the frequencies of interest, and the rigid barrier at the bottom of the tube is assumed to be a pressure-doubling surface. The wave propagation parameters required to fully characterize the acoustic properties of the porous medium are attenuation, phase velocity, complex intrinsic impedance and complex input impedance at the air/medium boundary. The phase velocity and the wavelength at a given frequency can, of course, be determined from one another. Also, the complex input impedance is completely dependent on the other propagation parameters. The wave propagation parameters can, in principle, be determined by choosing a Green’s function that represents both incident and reflected pressure waves in both the filled and unfilled portions of the tube, and fitting the unknown parameters of the Green’s function to the measured data by a least mean-squares estimation (LMSE) procedure. With 32 measured sound pressure values available it should be possible to determine the few unknown parameters (two real-valued parameters, i.e., the phase velocity and attenuation, and one complex-valued parameter, i.e., the reflection coefficient). The parameters of the air above the filled portion of the tube are presumed to be known.

In practice, the process does not work equally well for all parameters and for all frequencies and both types of gravel, and it has proven necessary to use a number of special procedures for the determination of some of the combinations. These procedures are described below and in more detail in [17]. In the case of the intrinsic impedance, no credible determinations have so far been achieved, though all other parameters can be reported with a satisfactory degree of confidence.

All measurements were taken in the impedance tube terminated with a heavy steel plate, massively stiffened, underlying a gravel layer of variable depth $L$. At the upper end of the impedance tube a loudspeaker was used to generate either a tone or band-limited random noise. The source frequency was kept low in order to propagate only plane waves normally incident upon the interfaces of air/gravel and gravel/steel plate. The transverse modes comprising the field around the loudspeaker are evanescent and strongly decaying. The $x$-axis has its origin at the gravel-backing interface (i.e., gravel/steel plate), and $x$ is increasing upward (see Fig. 3). For the fields below the speaker region within medium $0$ ($x > L$, Fig. 3. Schematic geometry of the experimental setup.)
air), medium 1 ($L \geq x > 0$, gravel) and at the bounding medium 2 ($x = 0$, rigid), the spectral sound pressure is

$$P(x, f) = \begin{cases} A_0 e^{i b_0 (x-L)} + \Gamma_{01} e^{i b_0 (x-L)}, & x > L \quad \text{(air)} \\ A_1 [p_0 e^{i b_1 (x-x_1)} + e^{i c_1 (x-x_1)}], & L > x > 0 \quad \text{(porous)} \end{cases}$$

(1)

where $\Gamma_{01}(f)$ is the reflection coefficient and $b_0(f)$, $b_1(f)$ are the attenuation and phase parameters, respectively. All of the above parameters depend on frequency $f$. As given, Eq. (1) describes a complex-valued spectral sound pressure, composed of incident (downgoing) and reflected (upgoing) waves, where $A_0$ has the magnitude and phase of the source. By fitting the measured rms sound pressure to $|P|$ of Eq. (1) through a LMSE process, estimates of acoustic propagation parameters, i.e., wavelength and attenuation, can be obtained.

The impedance $Z_2$ for the hard backing is assumed to be infinite, so the sound pressure is twice the incident value at the backing plate ($x = 0$). At the $01$ interface ($x = L$), the sound pressure solutions on either side must match in value, and the same condition applies to the particle velocity, yielding

$$A_0 (1 + \Gamma_{01}) = 2 A_1 \cos (b_1 - j x_1) L$$

(2)

These relations complete the specification of the spectral sound pressure within and above the porous layer.

### 3. Estimation of parameters

The rms sound pressures measured along the impedance tube vary with position, and the shapes of the standing wave profiles relate the propagation features involved. The shape of the standing wave profile (rms sound pressure vs. height) varies distinctly in response to depth of material, wavelength in the air, and within the material, and attenuation coefficient within the material. With sufficient data it should, in principle, be possible to estimate the few unknown wave parameters by a combined LMSE fit [18] of the Green’s function, Eqs. (1) and (2), to the measured sound pressure data. The fitting process seeks to minimize a residual error, chosen here as the mean of the squared differences between the measured rms sound pressures and the evaluation of $|P|$, at microphone positions $x_i$.

$$\delta^2 = N^{-1} \sum_{i=1}^{N} [P_{\text{meas, rms}}(x_i) - |P(x_i)|]^2$$

(3)

When evaluating $\delta^2$ of Eq. (3), a combination of values was chosen for the unknown parameters ($x_1$, $b_1$, $\Gamma_{01}$, and $A_0$), alongside a set of known values for parameters $\beta_0$ and $L$, all needed to completely specify $P(x)$ of Eq. (1). Under LMSE fitting, the unknown parameters were adjusted in combination, $\delta^2$ was evaluated, and the combination was reported that minimized $\delta^2$. When generating trial combinations, each parameter was assigned a plausible range and step size over which it was varied. For example, the attenuation coefficient was initially restricted to the range, $0 \leq x_1 \leq 10 \text{ Np/m}$, with 101 steps spanning the range. The real and imaginary parts of $\Gamma_{01}$ and $A_0$ were each treated as independent parameters with separately assigned ranges.

The LMSE adjustment of the parameters of the combined Green’s function, Eq. (1), to the measured sound pressure amplitude was performed using signals at the discrete frequencies 25, 50, 75, 100, 150 and 200 Hz. An example data set and fit is shown in Fig. 4. As mentioned above, with pressure amplitude data available at 32 points along the impedance tube, and with the known wave number in the air-filled portion of the tube, this proved to be difficult for some frequencies and some parameters. So, it was necessary in some cases to introduce analysis techniques tailored to specific parameters.

When the LMSE procedure does not converge to credible values of attenuation or phase velocity/wavelength, or when separate determinations are desired for reasons of confidence, more direct analysis techniques can sometimes be used. For example, phase velocity can be estimated from the time correlation of pressure signals between separated microphone ports; this appears to work best with band-limited random noise signals. The time delay for maximum correlation, divided into the linear distance between sampling points, yields the phase velocity. At frequencies for which the attenuation is high or at which multiple cycles of the standing wave exist in the porous medium, the exponent of a real exponential function can be fitted to the pressure standing wave data to determine the attenuation term of the complex wave number. Fitting to an exponential in this way amounts to replacing Eq. (1) by the expression $P(x) \approx A_1 \exp (x x_1)$ and neglecting the (presumed) smaller effects of the interference pattern. Both these latter methods agree well with results of the more general LMSE procedure at those frequencies at which both techniques are useful. While this paper represents a concise but sufficiently extensive presentation of the experimental measurements and data analysis procedures, additional details of the experiments and data analysis procedures are available elsewhere [17,19,20].

### 4. Results

The wave propagation parameters were estimated for two types of gravel: crushed limestone and pea gravel. The discussion in this section refers to five methods of estimating these parameters. The first, called the exponential fit, adjusts a single exponential $A_1 \exp (x x_1)$ to the pressure standing wave entirely within the porous medium to obtain $x_1$. The second method, the porous medium fit, considers only the standing wave in the porous medium and applies Eq. (1) for $L > x > 0$. The obtained quantities are $x_1$, $b_1$, $\Gamma_{01}$, and $c_1$. The third method, the air layer fit, applies Eq. (1) within the region $x \geq L$ to obtain $\Gamma_{01}$ and $Z_{01}$. The fourth method, the combined fit, applies Eq. (1) in both the porous layer and in the air, subject to the boundary condition of Eq. (2). In the combined fit the obtained quantities include $x_1$, $\beta_1$, $\lambda_1$, $c_1$, $\Gamma_{01}$, and $Z_{01}$. Finally, the correlation method refers to obtaining results for phase velocity $c_1$, and thus wavelength $\lambda_1$, via the cross correlation maximum as mentioned above.

#### 4.1. Crushed limestone

We tested Type CA-7 crushed limestone which is graded as follows: all particles are less than 25 mm in size, 50% are less than 12.5 mm, 7.5% less than 6.3 mm and 2% less than 1.8 mm.
The exponential fitting process (see Fig. 4). A plot of attenuation vs. frequency measured by two methods at different heights in the tube did not balance out. Thus, the porous medium fit is preferred for the 25- and 50-Hz attenuation estimates.

An obvious result in Fig. 5 is the discrepancy among the top, bottom and top-and-bottom attenuation curves. We have only speculations for this surprising result which has been confirmed by repeated measurements and a variety of computations. Although the crushed limestone is assumed to be of the same technical specification and was obtained from the same retail source, it was delivered in two batches at an interval of one year. The first batch, which was used to fill the lower half of the tube, may have had higher moisture content than the second batch, which filled the top half of the tube. In porous materials, the addition of even a small amount of water content dramatically changes the acoustic properties of the material. For example, an increase in water content has been observed to increase the attenuation of acoustic waves in sand [21,22]. Gravitational compression or settling may be involved. Complicating matters is the fact that sound amplitude correlation measurements made at different heights in the tube did not result in significant variations of phase velocity with height, with the exception of estimates at 150 Hz, as discussed below. One conclusion that can be made is that in such a poorly defined medium as crushed limestone substantial variations in sound propagation parameters may occur.

4.1.1. Wavelength

Fig. 6 illustrates the variation of wavelength in crushed limestone with frequency, as determined by fitting the parameters of the standing wave function to the measured amplitude data both in the porous medium alone (porous medium fit) and in the entire tube (combined fit). Specifically, best-fit values of parameters of $x_1$, $\beta_1$, $\Gamma_01$, and $A_0$ were selected by searching for the values that minimized the mean squared difference between the measured rms sound pressures and Eq. (1). From the estimate of $\beta_1$, the corresponding estimate of wavelength was then calculated from $\lambda_1 = 2\pi/\beta_1$.

In this case the data appear to be well-conditioned to estimate wavelength from parameter fitting, as the results are consistent between the two fitting procedures. Efforts were also made to determine phase velocity from correlation of amplitudes from pairs of microphone ports (using the relationship between wavelength and phase velocity), with mixed results. While the wavelength values agreed at the higher frequencies, at 25 and 50 Hz the correlation-derived values exceed the parameter-fitted values by about 40%. This results from the difficulty experienced by the correlation method in the presence of high standing wave ratios. This is confirmed by computing the phase velocity from the wavelength data in Fig. 6 and plotting the result on Fig. 7. Each individual data point in these two figures derives from a separate application of the LMSE algorithm. The close agreement of the three phase velocity curves over most of the frequency range suggests that the two outlier points on the “bottom of tube” curve resulted from fortuitous convergence of their respective LMSE runs to false minima in parameter space.

The apparent variation of attenuation with depth prompted comparisons between phase velocities measured only in the bottom of the tube to those only in the top (all within the porous layer). The results are given in Fig. 7. While the data are noisy, it
appears that, with the exception of 150 Hz, the phase velocity did not vary significantly with depth. This is confirmed by computing the phase velocity from the wavelength data in Fig. 6 and plotting the result on Fig. 7. Each individual data point in these two figures derives from a separate application of the LMSE algorithm. The close agreement of the three phase velocity curves over most of the frequency range suggests that the two outlier points on the "bottom of tube" curve resulted from fortuitous convergence of their respective LMSE runs to false minima in parameter space.

4.1.2. Input impedance

The magnitude of the reflection coefficient $|\Gamma_{01}|$ and its phase $\theta_{01}$ were obtained from fitting to Eq. (1). These results are shown in Table 1, for several frequencies and layer depths [19]. In Table 1, SWR is the pressure standing wave ratio in the air above the gravel column, and $Z_{01}/Z_0$ is the normalized input impedance. Input impedance is estimated by fitting the standing wave, including the reflection coefficient as a parameter, to the rms sound pressures in the air above the crushed limestone. Then, the input impedance is evaluated from $Z_{01} = Z_0(1 + \Gamma_{01})/(1 - \Gamma_{01})$, where $Z_0 = \rho_0 c_0$ is the intrinsic impedance of air. The results vary strongly with frequency as the pressure-doubling termination of the tube results in high standing wave ratio at the lower frequencies.

4.2. Pea gravel

We performed standard sieve analysis (USACE 1970, ASTM 2007) with the following results: A poorly graded non-plastic fine gravel (USCS: GP, AASHTO: A-1-a). Approximately 65% of the particles were between 12.7 mm and 6.4 mm, 28% between 4.76 mm and 6.35 mm and the remaining 7% were smaller than 4.76 mm.

The pea gravel is somewhat more homogeneous in particle size and is smaller in comparison with the diameter of a microphone port than is the crushed limestone. It follows that there should be fewer local distortions of the sound pressure fields in the neighborhood of microphone ports, thus fewer anomalous pressure data

![Fig. 7. Variation of phase velocity with frequency for crushed limestone. LMSE fits were performed separately for top and bottom halves of the porous layer [19]. The diamond-shaped data points were computed from the wavelengths in Fig. 6. The two outlier (square) points are believed to be erroneous.](image)

![Fig. 8. Attenuation coefficient in pea gravel estimated by the combined fit and the porous medium fit.](image)

![Fig. 9. Wavelength in pea gravel estimated by the combined fit and the correlation method. Due to high attenuation within the medium, the combined fit estimations have wide deviations at some frequencies. The correlation method proves to be very stable in the pea gravel since reflected waves are of low amplitude [19].](image)

![Fig. 10. Variation of phase velocity in pea gravel estimated by the correlation method.](image)
4.2.2. Wavelength results. As the attenuation is relatively high, the porous medium fit procedure gives the more consistent fit in the porous medium alone. As the attenuation is considerably greater than in the crushed limestone, the correlation method gives more repeatable results over three trials than does the combined fit method. The simple real exponential function proved more reliable, especially when the reflected wave was substantially reduced by attenuation. In some cases, the combined fit consistently yielded credible estimates of the wavelength and attenuation, estimates of the input impedance were very sensitive to small changes in the data. As an example, consider the estimated values of the acoustic parameters corresponding to Fig. 4 with best-fit values of 2.34 m, 0.49 Np/m, and 4.35 for the wavelength, attenuation coefficient and input impedance, respectively. If the data point corresponding to the microphone located at z = 1.6 m is removed, the LMSE remains the same but the best-fit parameters are 2.39 m, 0.47 Np/m, and 3.53 for the wavelength, attenuation coefficient and input impedance, respectively. This represents a change in the best-fit parameters of 2%, 4%, and 19% for the wavelength, attenuation coefficient and input impedance, respectively. While removal of one data point did not result in large changes in wavelength or attenuation estimates, significant changes were observed in the estimate of input impedance. Therefore, estimates of impedance appear to be very sensitive to measurement errors and less reliable. In those cases, restricting the fitting process to a smaller region of the tube sometimes yielded consistent results. In other cases, correlation methods or fitting to a simple real exponential function proved more reliable, especially when the reflected wave was substantially reduced by attenuation. In sum, for problems such as this, for which the measurements are inherently imprecise and in which several parameters are to be determined from the same data set, it may be necessary to attack the problem in a piecemeal way, choosing methods which are optimal for specific parameters rather than attempting to estimate all parameters at once by the combined fit process.

4.2.1. Attenuation

Fig. 8 gives the attenuation vs. frequency for the pea gravel as determined by both the combined fit to both media at once and the fit in the porous medium alone. As the attenuation is relatively high, the porous medium fit procedure gives the more consistent results.

4.2.2. Wavelength

Fig. 9 gives the wavelength vs. frequency for pea gravel computed by both the combined fit and the correlation methods. As the attenuation in this medium is considerably greater than in the crushed limestone, the correlation method gives more repeatable results over three trials than does the combined fit method.

4.2.3. Phase velocity

Fig. 10 shows the phase velocity vs. frequency within the pea gravel. Again, due to the high attenuation, the correlation method provides the most consistent results. The pea gravel did not suffer from the same variations in phase velocity with depth that was observed in crushed limestone.

4.2.4. Input impedance

Standing wave parameters calculated in the air region of the pea gravel tests are reported in Table 2. Where there were multiple datasets for the same gravel depth, the complex reflection coefficients were averaged and the SWR and input impedance were calculated from those averages. These input impedances were estimated from fitting to the “air” portion of the tube in the same way as for the crushed limestone.

5. Comments

Gravels are poorly defined media, with acoustical parameters that can vary significantly from sample to sample. While the gravels used by the construction industry are classified in ways that are apparently adequate for the purposes of that industry, those classifications probably only loosely constrain the acoustical parameters. Thus, numerous repetitions of experimental tests of the sort presented here, in order to increase the precision of parameter estimation, are probably not justified. The results presented herein can be considered representative but not definitive with respect to construction classifications.

The initial approach to the measurement of acoustical parameters was to construct a Green’s function representing an incident and a reflected wave of sound pressure in the tube, including both the gravel and the air above it, and to fit the parameters of the Green’s function to the measured amplitudes by the minimization processes described earlier. In several cases, the combined fit proved unable to yield credible estimates for one or more parameters, perhaps because the measurement errors by chance affected those particular parameters excessively. Specifically, while the combined fit consistently yielded credible estimates of the wavelength and attenuation, estimates of the input impedance were very sensitive to small changes in the data. As an example, consider the estimated values of the acoustic parameters corresponding to Fig. 4 with best-fit values of 2.34 m, 0.49 Np/m, and 4.35 for the wavelength, attenuation coefficient and input impedance, respectively. If the data point corresponding to the microphone located at z = 1.6 m is removed, the LMSE remains the same but the best-fit parameters are 2.39 m, 0.47 Np/m, and 3.53 for the wavelength, attenuation coefficient and input impedance, respectively. This represents a change in the best-fit parameters of 2%, 4%, and 19% for the wavelength, attenuation coefficient and input impedance, respectively. While removal of one data point did not result in large changes in wavelength or attenuation estimates, significant changes were observed in the estimate of input impedance. Therefore, estimates of impedance appear to be very sensitive to measurement errors and less reliable. In those cases, restricting the fitting process to a smaller region of the tube sometimes yielded consistent results. In other cases, correlation methods or fitting to a simple real exponential function proved more reliable, especially when the reflected wave was substantially reduced by attenuation. In sum, for problems such as this, for which the measurements are inherently imprecise and in which several parameters are to be determined from the same data set, it may well be necessary to attack the problem in a piecemeal way, choosing methods which are optimal for specific parameters rather than attempting to estimate all parameters at once by the combined fit process.

Acknowledgments

This work was conducted under Agreement No. W9132T-06-2-0002, Upper Mississippi Valley Cooperative Ecosystems Studies Unit., between the University of Illinois at Urbana-Champaign
and the US Army Construction Engineering Research Laboratory (CERL). The experiments are conducted on the premises of and with the facilities of CERL. The authors acknowledge the contributions of former Graduate Research Assistants Timothy Onder, Ryan Lee, Timothy Eggerding and Todd Borrowman, and of CERL employees Dr. Larry Pater, Niels Svendsen, Dr. Michelle Swearingen, Jeffery Mifflin, Robert Klein and Bruce MacAllister. Thanks are given to two anonymous reviewers whose questions prompted significant revision of the manuscript.

References


