

Erratum: “Frequency-dependent attenuation-compensation functions for ultrasonic signals backscattered from random media” [J. Acoust. Soc. Am. 111, 2308–2319 (2002)]

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The initial formulation of the problem in Ref. 1 did not separate out the coherent and incoherent scattering. While the form of the attenuation-compensation terms will not be affected, the error analysis was not complete. In reformulating the theory to separate out the coherent and incoherent scattering we start with revised Eqs. (10) and (11) from Ref. 1

$$r_{dx}(t) = s(t + 2d_1/c) + s(t + 2d_2/c) + s(t + 2d_3/c) + \dots \quad (10)$$

and

$$|R_{dx}(f)| = |S(f)| \times |e^{-i2\pi f(2d_1/c)} + e^{-i2\pi f(2d_2/c)} + e^{-i2\pi f(2d_3/c)} + \dots| \quad (11)$$

The second term on the right hand side of Eq. (11) simplifies as

$$|e^{-i2\pi f(2d_1/c)} + e^{-i2\pi f(2d_2/c)} + e^{-i2\pi f(2d_3/c)} + \dots|^2 = \sum_{i=j=1}^{\bar{n}} 1 + \sum_{i \neq j=1}^{\bar{n}} e^{-i2\pi f(2d_i/c - 2d_j/c)}, \quad (11b)$$

where the first term on the right hand side is the incoherent scattering term and the second term is the coherent scattering term, and \bar{n} is the number of scatterers in a unit length, dx .² Simplifying Eq. (11b) yields,

$$|e^{-i2\pi f(2d_1/c)} + e^{-i2\pi f(2d_2/c)} + e^{-i2\pi f(2d_3/c)} + \dots|^2 = \bar{n} + \Phi_{\text{coh}}. \quad (11c)$$

Relating Eq. (11c) to (11) gives

$$|R_{dx}(f)| = |S(f)| \bar{n}^{1/2} \left(1 + \frac{\Phi_{\text{coh}}}{\bar{n}} \right)^{1/2}. \quad (11d)$$

The term under the radical can be expanded as

$$\left(1 + \frac{\Phi_{\text{coh}}}{\bar{n}} \right)^{1/2} = 1 + \frac{1}{2} \frac{\Phi_{\text{coh}}}{\bar{n}} - \frac{1}{8} \left(\frac{\Phi_{\text{coh}}}{\bar{n}} \right)^2 + \dots = 1 + \Phi'_{\text{coh}}, \quad (11e)$$

where $\Phi'_{\text{coh}} < \Phi_{\text{coh}}$ for $\bar{n} \geq 1$. As \bar{n} (number density) becomes large, $\Phi'_{\text{coh}} \ll \Phi_{\text{coh}}$. Multiplying by the round trip propagation loss gives a revised Eq. (12) as

$$|R_{dx}(f)| = |S(f)| \bar{n}^{1/2} (1 + \Phi'_{\text{coh}}) e^{-2\alpha(f)x}. \quad (12)$$

Integrating Eq. (12) over the length of the gate, L , gives

$$|R_L(f)| = |S(f)| \bar{n}^{1/2} \frac{1}{L} \int_0^L (1 + \Phi'_{\text{coh}}) e^{-2\alpha(f)x} dx. \quad (13)$$

Equations (17)–(19) would then be rewritten as

$$|R_{dx}(f)|^2 = |S(f)|^2 \bar{n} (1 + \Phi_{\text{coh}}), \quad (17)$$

$$|R_{dx}(f)|^2 = |S(f)|^2 \bar{n} (1 + \Phi_{\text{coh}}) e^{-4\alpha(f)x}, \quad (18)$$

$$|R_L(f)|^2 = |S(f)|^2 \bar{n} \frac{1}{L} \int_0^L (1 + \Phi_{\text{coh}}) e^{-4\alpha(f)x} dx. \quad (19)$$

Omit (20). Equations (24)–(27) would then become

$$W_{OO}(f) = \frac{|P(f)|^2 |S(f)|^2}{L^2} \bar{n} \left[\int_0^L (1 + \Phi'_{\text{coh}}) e^{-2\alpha(f)x} dx \right]^2, \quad (24)$$

$$\epsilon_{OO}(f) = \frac{|P(f)|^2 |S(f)|^2}{L} \bar{n} \left(\frac{2}{A_{OO}^{1/2}} \int_0^L \Phi'_{\text{coh}} e^{-2\alpha(f)x} dx + \frac{1}{L} \left[\int_0^L \Phi'_{\text{coh}} e^{-2\alpha(f)x} dx \right]^2 \right), \quad (25)$$

and as αL becomes large

$$\epsilon_{OO}(f) = \frac{|P(f)|^2 |S(f)|^2}{L} \bar{n} \left(2 \int_0^L \Phi'_{\text{coh}} e^{-2\alpha(f)x} dx + \frac{1}{L} \left[\int_0^L \Phi'_{\text{coh}} e^{-2\alpha(f)x} dx \right]^2 \right), \quad (26)$$

$$\epsilon_{OM}(f) = \frac{|P(f)|^2 |S(f)|^2}{L} \bar{n} \int_0^L \Phi_{\text{coh}} e^{-4\alpha(f)x} dx. \quad (27)$$

As the number of scatterers, \bar{n} , increases the Φ'_{coh} term falls off much more rapidly than the Φ_{coh} term. Consequently, the error term for the Oelze and O'Brien compensation becomes

smaller relative to the error term from Eq. (27) for increasing \bar{n} .

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¹M. L. Oelze and W. D. O'Brien, Jr., "Frequency-dependent attenuation-compensation functions for ultrasonic signals backscattered from random media," *J. Acoust. Soc. Am.* **111**, 2308–2319 (2002).

²J. F. Chen, E. L. Madsen, and J. A. Zagzebski, "A method for determination of frequency-dependent effective scatterer number density," *J. Acoust. Soc. Am.* **95**, 77–85 (1994).