SCATTERER SIZE ESTIMATION USING A GENERALIZED ULTRASOUND ATTENUATION-COMPENSATION FUNCTION TO CORRECT FOR FOCUSING

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Abstract - The frequency statistics of the backscattered waveform observed between tissue boundaries can be matched to a model to estimate the characteristic size of the scatterer in the tissue region of interest. In the past, the models have all assumed plane wave propagation requiring the use of unfocused or weakly focused sources in the experiment. In our work, we challenged this assumption by re-deriving the scattering equations assuming that the focused field can be accurately modeled as a 3D Gaussian distribution in addition to the traditional assumptions that the scatterers are a sufficient distance from the source and that the field is approximately constant across the scatterer. We showed that correcting for focusing when estimating the scatterer size only requires a generalized attenuation-compensation function that includes the impact of focusing along the beam axis. We then generated computer simulations to aid in the understanding of the impact of diffraction along the beam axis and compared the performance of our attenuation-compensation function to the performance of previously proposed attenuation-compensation functions for various levels of focusing (*f*/1, *f*/2, *f*/4), attenuation (0.05 to 1 dB/cm/MHz), and rectangular window length used to gate the time domain signal (1 to 13 mm). The generalized attenuation-compensation function yielded results accurate between 2% and 7.2% while the traditional attenuation-compensation functions that neglected focusing had errors greater than 50% for moderate attenuation and window lengths.

I. INTRODUCTION

One of the distinguishing features of any ultrasound image of biological tissue is the scattering observed between tissue boundaries. The frequency statistics of this backscatter contains information related to the tissue microstructure (i.e. characteristic size of the scatterers). In order to extract the microstructure information, models have been developed to elucidate scattering details.

Most of the previous models have assumed plane waves incident on the scattering region while only considering diffraction effects in the transverse plane [1,2]. Diffraction effects along the beam axis have been neglected. Other authors included a complete Green's function description of the source when determining the scattered field [3]. Unfortunately, the resulting equations were cumbersome requiring a precise knowledge of the source's excitation in order to solve for the required fields. As a result, it is difficult to use their results when experimentally calibrating a focused source. Hence, most investigators only use large f-number transducers in their backscatter analyses. This is potentially restrictive in diagnostic imaging systems where smaller f-numbers may be desirable to improve the spatial resolution laterally.

In this paper, we re-derive the expected backscattered signal from a region of randomly positioned identical scatterers without the plane wave approximation. Instead, we assume that the field near the focus can be accurately modeled as a three-dimensional Gaussian beam while continuing to assume that the scatterers are a sufficient distance from the source, and the field is approximately constant across the scatterer. We show that correcting for focusing when estimating the scatterer size only requires a generalized attenuation-compensation function that includes the impact of focusing along the beam axis. We then generate computer simulations to aid in the understanding of the impact of diffraction along the beam axis and compare the performance of our attenuation-compensation function to the performance of previously proposed attenuation-compensation functions for various levels of focusing (*f*/1, *f*/2, *f*/4), attenuation (0.05 to 1 dB/cm/MHz), and rectangular window length (1 to 13 mm).

II. THEORETICAL DERIVATIONS

In this section of the paper, the major assumptions involved with the theoretical derivation of the backscatter voltage for a focused source are highlighted.

The coordinate system for the derivations is shown in Figure 1.

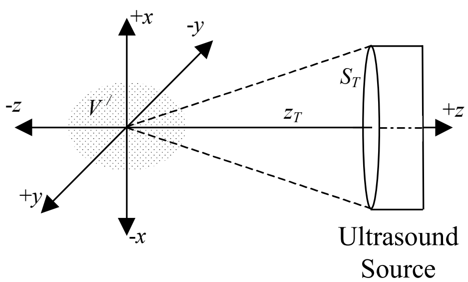

Figure 1: Coordinate system for derivations.

In this figure, *V'* is the region of the tissue about the focus where the characteristic size of the tissue microstructure is to be estimated, S_T is the aperture plane of the ultrasound source, and z_T is the distance along the beam axis from S_T to the focus. The origin of the coordinate system has been placed at the focus. We also relate the particle velocity, u_z, on S_T to the voltage of the ultrasound source at frequency f according to

$$u_z(\vec{r}_T, f) = K_{uV}(f) V_{inc}(f) G_T(\vec{r}_T, f) H(f)$$
$$V_{refl}(f) = \frac{H(f)}{S_T K_{uV}(f)} \iint_{S_T} d\vec{r}_d \, u_z(\vec{r}_d, f) G_T(\vec{r}_d, f) \quad (1)$$

where K_{uV} is a conversion constant relating voltage to particle velocity, V_{inc} is the voltage applied to the ultrasound source during transmit, H is the filtering characteristics for the ultrasound source, G_T is a dimensionless aperture gain function that accounts for focusing, and V_{refl} is the voltage returned from the ultrasound source by the scattering.

In the course of the derivation, we make the traditional assumption that the dimensions of the scattering region, *V'*, are small compared to its distance from the source and detector and that the field is constant across each individual scatterer [3]. Based on these assumptions, the pressure experienced by each scatterer at location $\vec{r}\,'$, neglecting multiple scattering, is given by

$$p_{inc}(\vec{r}\,', f) \cong -if\rho K_{uV}(f) V_{inc}(f) H(f) \cdot \iint_{S_T} G_T(\vec{r}_T, f) \frac{e^{i\tilde{k} r_T - i\tilde{k} \frac{\vec{r}_T}{r_T} \cdot \vec{r}\,'}}{r_T} d\vec{r}_T \quad (2)$$

where $\tilde{k}$ is the complex wave number and ρ is the density of the tissue. Likewise, the backscattered voltage returned by the source is given by,

$$V_{refl}(f) \cong -\frac{2i V_{inc}(f) H^2(f) \tilde{k}^3}{(4\pi)^2 S_T} \cdot \iiint_{V'} d\vec{r}\,' \left[\gamma(\vec{r}\,') \left(\iint_{S_T} d\vec{r}_T G_T(\vec{r}_T, f) \frac{e^{i\tilde{k} r_T - i\tilde{k} \vec{r}\,' \cdot \frac{\vec{r}_T}{r_T}}}{r_T} \right)^2 \right] \cdot \quad (3)$$

In order to simplify the analysis and obtain closed form expressions, we further assume that in the focal region

$$\iint_{S_T} d\vec{r}_T \left(G_T(\vec{r}_T, f) \frac{e^{i\tilde{k} r_T - i\tilde{k} \vec{r}\,' \cdot \frac{\vec{r}_T}{r_T}}}{r_T} \right) \cong \left(G_o \, e^{-\left(\left(\frac{x'}{w_x}\right)^2 + \left(\frac{y'}{w_y}\right)^2 + \left(\frac{z'}{w_z}\right)^2 \right)} \right) e^{i\tilde{k}(z_T - z')} \quad (4)$$

where w_x, w_y, and w_z are the frequency dependent equivalent Gaussian beamwidths in the focal plane and along the beam axis, and G_o is the geometric gain value for the pressure field at the focus.

With this Gaussian approximation, the expected value for the backscattered power spectrum is given by,

$$\mathrm{E}\left[\left| V_{refl}(f) \right|^2 \right] \propto \frac{w_x w_y |k|^6 |H(f)|^4 |V_{inc}(f)|^2}{A_{comp}(f)} \Re_{\gamma\gamma}(2k\hat{z}) \quad (5)$$

where $\Re_{\gamma\gamma}$ is the Fourier transform of the spatial autocorrelation function for the scatterer given by [2]

$$\Re_{\gamma\gamma}(2k\hat{z}) \propto e^{-0.827 k^2 a_{eff}^2} \quad (6)$$

for scatterers with Gaussian impedance distributions where a_{eff} is the effective radius of the scatterer. Also, A_{comp} is a generalized attenuation-compensation function that corrects for both focusing and attenuation and is given by,

$$A_{comp}(f) = \frac{e^{4\alpha_{eff} z_T}}{\left(\int_{-L/2}^{L/2} ds_z g_{win}(s_z) e^{-4\left(\frac{s_z^2}{w_z^2}\right)} e^{4\alpha s_z} \right)} \quad (7)$$

where L is the length of the window, g_{win}, used to gate the time domain signal (i.e. length of time gate, T_{win}, given by $T_{win} = 2L/c$ where c is the sound speed of the tissue), α is the attenuation in the scattering region, and α_{eff} is the effective attenuation along the beam axis going from the source to the scattering region.

III. OTHER COMPENSATION FUNCTIONS

Point Compensation

If we neglect all field variations in the scattering region, the integral in the denominator of Equation (7) drops out and A_{comp} becomes,

$$A_{PC}(f) = e^{4\alpha_{eff} z_T} / L \quad (8)$$

which is the traditional point compensation for attenuation [4]. Point compensation should over compensate when the attenuation in the scattering region is dominate (i.e. $1/w_z << \alpha$) resulting in an underestimate of the scatterer size. Likewise, point compensation should under compensate for attenuation when the focusing is dominate (i.e. $1/w_z >> \alpha$) resulting in an overestimate of the scatterer size.

O'Donnell-Miller Compensation

Similarly, if we use a rectangular window to gate the time domain signal and let $w_z \to \infty$ (i.e. unfocused source), then Equation (7) becomes,

$$A_{OM}(f) = \frac{4\alpha e^{4\alpha_{eff} z_T}}{e^{4\alpha\frac{L}{2}} - e^{-4\alpha\frac{L}{2}}} \quad (9)$$

which is the O'Donnell-Miller attenuation-compensation function [5]. A_{OM} should always under compensate resulting in an overestimate of the scatterer size.

Oelze-O'Brien Compensation

Another attenuation-compensation function found in the literature is the Oelze-O'Brien attenuation-compensation function given by,

$$A_{OO}(f) = \frac{e^{4\alpha_{eff} z_T} e^{-4\alpha\frac{L}{2}}}{L} \left(\frac{2\alpha L}{1 - e^{-2\alpha L}} \right)^2 . \quad (10)$$

A_{OO} cannot be derived from Equation (7).

IV. SIMULATION ANALYSIS

In order to compare the different attenuation-compensation functions, computer simulations were generated modeling the backscatter returned from a focused source sonifying an infinite half-space containing Gaussian scatterers. The attenuation of the half-space was varied from 0.05 to 1 dB/cm/MHz in different simulations and had a sound speed of 1532 m/s. The scatterers had an effective radius of 25 μm and were randomly placed in the focal region at a density of 35/mm^3. For each value of half-space attenuation, 1000 independent distributions of scatterers were generated and windowed with rectangular window lengths from 1 to 13 mm. These distributions were then averaged in the frequency domain in groups of 25 and compared to a reference signal from a rigid plane at the focal plane to obtain 40 estimates of the scatterer size for each of the attenuation compensation functions [2].

The source used in the simulations had a focal length of 5 cm and H given by,

$$H(f) \propto f \cdot \exp\left(-\left(\frac{f - 8\,\text{MHz}}{6\,\text{MHz}} \right)^2 \right) \quad (11)$$

which is similar to a PZT spherically focused transducer in our lab. The Gaussian fields in the focal region were given by,

$$w_{x,y} = 0.87\lambda f\# \quad w_z = 6.01\lambda (f\#)^2 \quad (12)$$

where λ is wavelength. The Gaussian beamwidths were found by matching the –3-dB transmit beamwidth for an ideal spherically focused source.

The simulation results for each of the compensation functions for transducers with f-numbers of 1, 2, and 4 at a half-space attenuation of 0.5 dB/cm/MHz are shown in Figure 2. Notice that for smaller window lengths all of the compensation functions give comparable performance. However, at larger window lengths, only A_{comp} gives accurate scatterer size estimates.

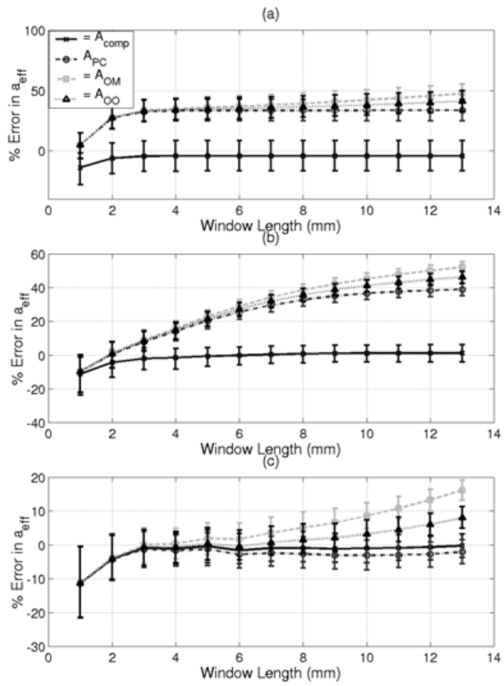

Figure 2: Results for (a) *f*/1, (b) *f*/2, and (c) *f*/4 at half-space attenuation of 0.5 dB/cm/MHz.

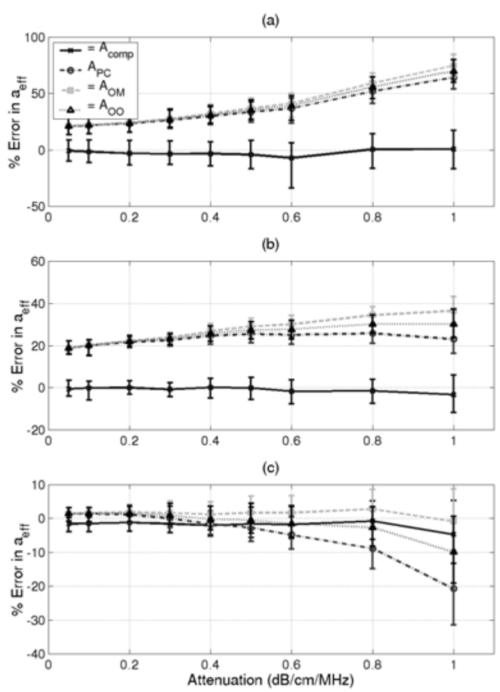

Figure 3: Results for (a) *f*/1, (b) *f*/2, and (c) *f*/4 at a window length of 6 mm.

The accuracy of A_{comp} over the other attenuation-compensation functions is further illustrated in Figure 3 that shows the simulation results for all half-space attenuations using a window length of 6 mm. Also, notice from these plots that when focusing dominates (i.e. *f*/1 and *f*/2 sources), all of the traditional attenuation-compensation functions overestimate the scatterer size. However, when the attenuation dominates (i.e. *f*/4 source), point compensation underestimates the scatterer size while O'Donnell-Miller still overestimates the scatterer size as was expected from our theoretical analysis. Oelze-O'Brien compensation falls between the other two traditional compensation functions.

V. ACKNOWLEDGMENTS

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VI. REFERENCES

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