Application of an acoustic backscatter technique for characterizing the roughness of porous soil

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An acoustic backscatter technique proposed by Oelze et al. [J. Acoust. Soc. Am. 109, 1826–1832 (2001)] was used to characterize the roughness of porous soil surfaces. Roughness estimation errors are minimized when the effective flow resistivity of the porous soil is high, e.g., above 300,000 mks Rayls/m. Four soil plots were constructed by roughening soil with farming implements. Three plots were sealed using Saran powder dissolved in methyl ethyl ketone (MEK) and then covered to prevent further weathering. A fourth plot was left in the open and exposed to rainfall, which also acted to seal the surface and further change the roughness. In sealing the surface the effective flow resistivity of the surface was increased above 300,000 mks Rayls/m, which is typical for weathered agricultural surfaces. The roughness power spectra of the soil surfaces were measured by acoustic backscatter and alternatively by a laser profiler. Regression analysis was used to approximate each roughness power spectrum versus roughness wave number with a best-fit line. The best-fit line was used to calculate the rms height and the correlation length of the rough surface by integrating the approximate roughness power spectrum over a range of roughness wave number values. The range of roughness wave number values defines the roughness length scales used in the statistical calculations. High-roughness wave numbers correspond to smaller length scales of roughness and low-roughness wave numbers correspond to larger length scales of roughness. Over certain ranges of roughness wave number values the statistics from the acoustic backscatter and laser profiler measurements is in good agreement. However, as the low-cutoff roughness wave number is decreased and the high-cutoff roughness wave number is increased, agreement between the laser and acoustic techniques diminishes. © 2002 Acoustical Society of America. [DOI: 10.1121/1.1459462]

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I. INTRODUCTION

Quantifying the roughness of agricultural surfaces is important to determine the degree of local soil erosion through erosion models and evaluate the effects of tillage processes. Roughness is important to erosion control of agricultural surfaces since it impedes water run-off. If a fallow agricultural field is left with no roughness, rainfall can wash away the topsoil into local rivers and streams. Topsoil is vital to growing crops and once the topsoil is washed away, it is difficult if not impossible to replace. Roughness prevents the washing away of topsoil because the valleys and dips on a rough surface allow the water from rainfall to pool.

Much work has been done to examine the effects of roughness on soil erosion.1–4 For soil conservation use it is important to have the ability to rapidly quantify the roughness of agricultural lands. Relating the roughness statistics to erosion allows for farmers or other soil conservationists to keep fallow agricultural surfaces sufficiently roughened to prevent soil erosion.

Several instruments have been used in the past to obtain the surface roughness statistical characterizations. Contact-type microreliefmeters have been used to measure surface roughness.5,6 Contact methods tend to be time-consuming, cumbersome, and have the great disadvantage of disturbing or damaging the soil surface. Römkens and Wang first described the use of a noncontact, laser microreliefmeter at the Fourth Federal Interagency Sedimentation Conference.7 A subsequent paper described in detail the operation and performance of the laser microreliefmeter.8 The laser profiler has been used to study the relationship of roughness to soil erosion and profile evolution with increased rainfall and tillage.4,7,9 A diagram of the laser microreliefmeter is shown in Fig. 1.

The laser microreliefmeter effectively measures the surface statistics of rough soil surfaces without disturbing the soil surface. However, the equipment is expensive, somewhat bulky, and scan times can take several hours for a plot of 60×60 cm. An alternate noncontact method was proposed by Oelze et al. using acoustic backscatter techniques to measure surface roughness statistics.10 The goal was to find a quick and economical means to noninvasively evaluate surface roughness statistics.

Surface roughness characterization by the use of acoustic backscatter has been used for many years in the underwater acoustics community. A review of acoustic backscatter to characterize surface roughness in underwater sound is presented in Jackson et al.11 Use of an acoustic backscatter technique to measure surface roughness offers the possibility of

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an inexpensive and quick method of determining surface roughness statistics.

Acoustic backscatter techniques were proposed because of the limitations of acoustic forward-propagation techniques. Previous models of acoustic forward-propagation techniques did not allow the measurement of the roughness of porous soil independently from the effects of the pore properties of the soil. Acoustic forward-propagation techniques have been used invasively and noninvasively to measure the pore properties of soils. Invasive techniques, like the probe microphone technique, examine sound as it propagates through soil by measuring the magnitude and phase of sound.12–16 From the phase and magnitude of the sound as it propagates, the complex acoustic wave numbers for the porous material are obtained. Measured propagation wave numbers are related to pore properties of the soil.

Noninvasive techniques, such as the level difference technique, look at sound as it reflects from the porous surface.12,13,17,18 Impedance values measured by the reflection coefficient from the surface at different frequencies are used to estimate pore property values. The level difference technique was able to quantify the pore properties of porous soil surfaces over smooth, flat ground. Roughness of the soil surface complicated measurements of the pore properties by mimicking the impedance contribution of the pore properties.19,20 Previous models showed that separating the roughness effect from the pore property effect was impossible.20,21 Combining the probe microphone technique with the forward-scattering technique allowed the separation of pore property effects from roughness effects. The previous models used to describe the roughness effect on acoustic forward-propagation measurements combined vertical and horizontal roughness statistics into an effective ground impedance. According to the previous theory, however, horizontal (spatial) and vertical (height) roughness statistics could not be separately obtained with acoustic forward-propagation techniques.

Recent models of acoustic forward-propagation techniques describe the roughness with separable horizontal and vertical roughness parameters.22,23 Furthermore, the recent models suggest that the roughness parameters may be separable from the pore property effects of the soil. The recent acoustic forward-propagation models are sensitive to the porosity of the surface. Inversion of the new forward-propagation models to extract roughness parameters from pore properties (porosity) is possible but has not been done. If the measurement is more sensitive to porosity than to the roughness, estimation of roughness parameters may be inaccurate. The recent models allow for separation of the roughness effects from the pore property effects, but the accurate estimation of these parameters using the model still needs to be tested.

The acoustic backscatter technique does not suffer from the same limitations as the acoustic-forward propagation techniques. Oelze et al. showed theoretically that acoustic backscatter could independently measure roughness from pore property effects in typical weathered agricultural surfaces.10 The acoustic backscatter also gives two independent parameters describing the roughness. The first parameter describes roughness on the vertical scales (rms height) and the other parameter describes the spatial distribution of roughness (correlation length of the roughness). Experiments were conducted that showed the sensitivity of acoustic backscatter to different roughness sizes and determined the rms height and correlation length statistics.24

In this paper the results of further acoustic backscatter experiments are examined for their ability to accurately predict roughness statistics of typical weathered agricultural surfaces. The calculated roughness statistics are affected by the choice of roughness wave number range used. The low-cutoff roughness wave number determines the largest scales of roughness that will be incorporated in the statistical calculation. The high-roughness wave number limits the smallest roughness scales that will be included in the statistical calculations. Actual acoustic measurements are examined to determine the effects of low- and high-roughness wave number cutoff choices on the statistical calculation. Section II describes the acoustic backscatter and laser profiler devices and preparation of the experimental roughness plots. Section III describes the theory used to obtain the roughness statistics from the acoustic and laser measurements. Section IV lists the experimental results of measurements of rough surfaces by the acoustic backscatter and laser profiler technique and examines the choice of roughness wave number cutoff on the statistical calculations of the rms height and correlation length.

II. ACOUSTIC BACKSCATTER AND LASER PROFILER DEVICE

A. Acoustic backscatter device

Acoustic backscatter techniques relate measurements of the intensity of backscattered sound from a rough surface to models describing the surface properties. Figure 2 shows a typical experimental setup for an acoustic backscatter measurement from a rough surface. A transducer is positioned at some height, h, above a rough surface directed at a grazing angle, θg, towards the surface. The graze angle, θg, is measured from a plane parallel to the surface. A pulse or tone burst of sound is emitted from the transducer and propagates towards the surface. The transducer is operated in pulse/echo mode so that it is used not only as the transmitter but also as the receiver. The sound that scatters back towards the transducer is called the backscatter. When the size of the transducer aperture is large relative to the wavelength of sound, the sound emitted is in an acoustic beam. A beam enables the
sound to be localized to a specific spot on the surface. In Fig. 2, $A$ is the area ensonified on the surface and is the projection of the acoustic beam on the surface.

Acoustic backscatter is measured in terms of the scatter strength, with units in decibels (dB). The backscatter strength is defined as

$$S_s = 10 \log (\sigma),$$

where $\sigma$ is the backscattered cross section. The backscattered cross section is given by

$$\sigma = \frac{r^2 I_s}{I_0 A},$$

where $I_s$ is the intensity of the field scattered back towards the source, $r$ is the axial distance from the source to the ensonified area, $I_0$ is the incident intensity, and $A$ is the area of the ensonified surface.

When sound is incident on a rough soil surface with graze angle, $\theta_g$, most of the sound is reflected away from the transducer at an angle, $\theta_g$. If the graze angle is large, some of the sound will be received by the transducer beam. For very low graze angles, the transducer will receive none of the sound reflected from the surface. As shown in Fig. 3, a small portion of the sound is scattered from the roughness in all directions. The sound level of the source must be large enough so that the portion of sound scattered from the surface back to the transducer is detectable above the noise. The sound intensity level (SIL) is a decibel scale, and is defined as

$$\text{SIL} = 10 \log \frac{I_s}{I_{ref}},$$

where $I_s$ is the intensity of the sound source at 1 m and $I_{ref}$ is a reference intensity for sound in air equal to $10^{-12} \text{ W/m}^2$. Since the scattering returns are usually on the order of 20 to 30 dB less than the incident sound intensity level, it is necessary to have a source level at least 40 dB above the noise at the surface. Noise levels of sound outdoors are typically between 50–70 dB for the frequency ranges of 1–10 kHz, the frequencies used in the experiments. The source levels of transducers used for roughness characterization of porous soils outdoors should be at least 100 dB in order to discern the scattered signal.

The scattered intensities are measured and inserted into Eq. (2) to calculate the backscatter cross section. The effect of the equipment (i.e., the magnitude of the output and receiving sensitivity of the transducer) on the scattered returns is taken out by the substitution method. The same transducer used in the scattering experiment is used to measure the reflection of the pulse from a planar surface at normal incidence. The planar surface in this work was a large sheet of Plexiglas that was assumed to be a perfect reflector. The reflected pulse is used as a reference for the incident pulse. Since the same equipment and settings were used for the incident and scattered intensities, the effect of the equipment on the measured cross section is canceled out.

For the acoustic backscatter measurements taken on the four roughness plots, a capacitor style transducer with grooved backplate covered by Mylar film was used. The transducer was circular, with a diameter of around 25 cm and a flat frequency response over the range of 1 to 10 kHz. The transducer was used in pulse/echo mode through a diode clamp over the frequency range of 1.5 to 10 kHz. Two- or three-cycle tone bursts were generated by an HP model 3314A function generator and amplified to the desired voltage. A 200-V dc bias was placed on the transducer using a Heathkit model IP-2717A power supply. Backscatter signals were received by the same transducer and filtered and amplified by an SRS 650 filter. The resulting signals were displayed on an HP model 54540C oscilloscope and saved to a disk for postprocessing.

For the measurements the transducer was mounted to a movable cart. The mounting bar could be adjusted to change the elevation of the transducer element with respect to the surface. The screw attaching the transducer to the mounting bar could adjust the graze angle of the transducer. Figure 4 shows a diagram of the cart used to mount the transducer. The cart was rolled down the length of a soil plot, allowing measurements to be taken at different points on the surface for averaging.

**B. Laser profiler technique**

The roughness of the soil plots were also measured by a laser profiler (see Fig. 1). The laser profiler has been used in the past to measure the roughness of soils. The laser profiler is a nonacoustic technique and is not affected by the pore properties of the soil.

The laser profiler works by focusing a laser beam on the rough surface. A lens attached to the laser box passes light to a detector that converts the light to a voltage. The higher up
the light appears on the detector screen, the lower the voltage, which in turn corresponds to a lower roughness height. The laser profiler works by converting the voltage level to a roughness height. For the detector used in the experiment, 1 volt corresponded to 1/2 cm. Potential for errors occur with the laser profiler if the rough surface blocks the laser spot from the detector. The larger the roughness, the greater the potential for the roughness to block the view of the detector. Measurements with error due to rough surface blockage were not included in the data.

The laser profiler was mounted on a table situated on a metal cage that sat on tracks attached to an aluminum frame. Figure 5 shows a diagram of the laser profiler mount. The tracks allowed the laser profiler to translate in one horizontal direction. The table mount was attached to rails and a screw that went across the length of the cage. The screw was attached to a step motor that allowed for the laser/detector apparatus to translate in a horizontal direction perpendicular to the direction of the tracks. The aluminum frame sat on six legs that allowed for the level of the frame to be adjusted. Leveling is done in order to take out any slant that would be detected as long wavelength roughness.

Transects 60 cm in length were measured from the surface in the horizontal direction perpendicular to the tracks (x direction). Each transect was taken by recording the laser profiler readings at certain time intervals as the motor translated the profiler in the x direction. The motor was controlled by a computer program that read in the values obtained by the laser profiler. After a full translation in the x direction, the laser profiler was then translated 3 mm in the y direction by physically moving the cage on the aluminum frame tracks forward by 3 mm. Once the cage and laser profiler were translated forward, the motor then translated the profiler in the x direction another full turn, 60 cm. The process was repeated until a profile with area of 60×60 cm was measured.

Data acquisition was accomplished by use of a PC, an HP 3478A multimeter, a GP–IB interface card and cable, and an optocator. The optocator delivers a dc voltage from the laser detector to the multimeter that in turn is sent to the computer through the GP–IB interface. The value measured by the multimeter is sent to the computer, which saves the value in a sequential array to the PC’s hard drive. The spacing between each point is determined by the speed of the motor, i.e., the slower the motor the more points are obtained. The distance between each point can be calculated by dividing the total translated distance by the total number of recorded points. After measuring each transect, a matrix of 60×60 height values is obtained. This matrix or area profile, \( h(x,y) \), can be broken into separate x and y transects.

From a height profile transect in the x- or y direction obtained by the laser profiler, the 1D power spectrum can be calculated through the equation

\[
W(k) = \frac{1}{2\pi L} \left| \int_0^L h(x_i) e^{ikx_i} dx_i \right|^2,
\]

where \( L \) is the length of the transect and \( x_i \) represents the 1D transect direction. The total 1D power spectrum is calculated by averaging the 1D power spectra from transects recorded in the x- and y directions. Averaging allows for the total 1D power spectrum to be smoothed and to decrease the effects of anomalous roughness.

If the surface is isotropic in the x- and y-directions, then the roughness power spectra from the x- and y-transects will have approximately the same form. For truly random rough surfaces, the assumption of isotropy will be true. In order to compare the total 1D power spectrum measured by the laser profiler with the 2D power spectrum obtained by the acoustic backscatter technique, the total 1D power spectrum is converted to a 2D power spectrum. Under the assumption of isotropy and that the roughness power spectrum for soil surfaces is a power law and fractal in nature, the 1D power spectrum can be related to the 2D power spectrum. Assuming a power-law form and fractal nature of the surface, then the 2D power spectrum is given by

\[
W(k_x,k_y) = \frac{\beta}{(k_x^2 + k_y^2)^{\gamma/2}},
\]

where \( \beta \) is the 2D intercept and \( \gamma \) is the power-law exponent with the constraint \( 2 < \gamma < 4 \). The 2D power spectrum intercept and exponent can be related to the intercept and exponent of the 1D power spectrum by integrating Eq. (5) over one variable. Integrating Eq. (5) over one variable gives
III. THEORY

C. Plot preparation

Acoustic backscatter experiments were conducted in situ on rough soil surfaces. The 1.5-m×4-m plots were constructed by breaking up the soil with various farming implements, i.e., a hoe and rake. Clods with radius up to 8 cm were present on the surfaces after preparation. The first three soil surfaces were coated with powder Saran dissolved in MEK as measured by the laser profiler.

From a 1D power-law power spectrum that is isotropic, the approximate 2D roughness power spectrum can be obtained by converting the 1D intercept and power exponent into the 2D intercept and power exponent through Eq. (8).

\[
W(k)dk = \beta \frac{1}{k^{\gamma - 1}} \text{dk}, \quad \beta = \frac{(2 \pi)^{\gamma - 2} \phi \Gamma\left(\frac{\gamma}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{1 - \gamma}{2}\right)} \text{ and } m = \gamma - 1.
\]

From a 1D power-law power spectrum that is isotropic, the approximate 2D roughness power spectrum can be obtained by converting the 1D intercept and power exponent into the 2D intercept and power exponent through Eq. (8).

The fourth plot was constructed in a similar fashion to the medium-sized roughness plot. Figure 7 shows a random cross-sectional transect of the fourth roughness plot. Unlike the first three plots, the fourth plot was left open and exposed to a rainstorm. The rain served two purposes in that it further broke down the roughness clods and decreased the acoustic permeability (raised the effective flow resistivity). After being exposed to the rainstorm, the surface was covered to prevent further erosion while the experiment was being conducted. The flow resistivity was measured independently by a Leonard’s apparatus and shown to be above \(3 \times 10^5\) mks Rayls/m for each plot.

III. THEORY

In this work, the roughness of a soil surface is modeled as a smooth surface with small perturbations. The perturbation theory for a rough surface assumes the roughness and local slope is small compared to the acoustic wavelength. Acoustic backscatter from rough surfaces can be modeled using first-order perturbation theories at a rough fluid–fluid interface. First-order perturbation theory is valid under the constraints that

\[ k_d h \sin \theta_s < 0.5. \]

where \(h\) is the rms height of the surface, \(k_d\) is the acoustic wave number, and \(\theta_s\) is the graze angle. The first-order backscatter cross section is given by

\[ \sigma_{1}(\theta_s) = 4k_d^2 \sin^2 \theta_s |Y(k_d, \theta_s)|^2 W(2k_d \cos \theta_s, 0), \]

where \(W(2k_d \cos \theta_s, 0)\) is the 2D roughness power spectrum evaluated at the Bragg wave number. The 2D roughness power spectrum, \(W(K)\), represents the scattered power at a roughness wave number defined by \(|K| = 2k_d \cos \theta_s\) in Eq. (10). The middle term in Eq. (10)

\[ Y(k_d, \theta_s) = \frac{(\rho - 1)^2 \cos^2 \theta_s + \rho^2 - \kappa^2}{[\rho \sin \theta_s + \sqrt{\kappa^2 - \cos^2 \theta_s}]^2}, \]

is a modified reflection coefficient, \(\rho\) is the ratio of the density of the surface material to the density of air, and \(\kappa\) is the ratio of rough surface wave numbers to air wave numbers. Substituting the complex wave number and complex density from Attenborough’s analysis of rigid frame porous materials yields the modified reflection coefficient in terms of the surface impedance.
where $k_b$ is the complex wave number of the porous soil surface and $Z$ is the complex acoustic impedance of the soil surface.

The acoustic surface impedance and complex wave number describing a porous soil are functions of the frequency of sound and the pore properties of the soil surface. The acoustic impedance and complex wave number describe the interaction of sound with the soil. For very large surface impedance the sound will be perfectly reflected from the surface. For a finite surface impedance the amount of sound absorbed by the surface will depend on the frequency of sound and pore properties of the soil.

The pore properties that influence the propagation of sound into and through the soil are the porosity, tortuosity, and effective flow resistivity. The porosity is a unitless parameter and is the ratio of the open space in the soil relative to the overall soil volume. The tortuosity is the effective length of the pore per unit depth and describes the amount of crinkle and interconnectivity of the pore pathways in the soil. The effective flow resistivity measures the permeability of the soil to sound. A high flow resistivity means that the surface becomes more impenetrable to sound, or acoustically harder. The value of porosity is always less than 1, the tortuosity is less than 10, and the effective flow resistivity can range from $10^0$ to $10^6$ (mks Rayls/m). For acoustically harder surfaces (effective flow resistivity greater than $3 \times 10^5$ mks Rayls/m) the impedance is dominated by the flow resistivity term and the other pore properties have negligible effect. Furthermore, the response of the complex surface impedance to changes in flow resistivity for acoustically harder surfaces is very small.  

For acoustically harder surfaces, such as weathered agricultural surfaces, assuming a value for the effective flow resistivity of $1 \times 10^6$ yields the complex surface impedance and wave number with minimal error. The contribution of the modified reflection coefficient to the acoustic backscatter strength measurement can be calculated from the approximated values of the complex surface impedance and wave number. The 2D roughness power spectrum can then be determined at a particular frequency and graze angle by relating an acoustic backscatter strength measurement with the theoretical perturbation cross section, Eq. (10).

10 log $W(2k_g \cos \theta_g,0) = S_f(k_a, \theta_g) - 10 \log [4k^4_\alpha \sin^4 \theta_g |Y(k_a, \theta_g)|^2].$  

By varying the frequency of operation (acoustic wave number) and graze angle, different portions of the roughness power spectrum, $W(k_a, \theta_g)$, may be mapped out.

Random rough surfaces tend to have roughness power spectra with power-law dependence. Soil plots broken up with farming implements were also shown to have power-law dependence. When plotting a power spectrum that is a power law in log–log space, the power spectrum is a line whose slope is the power-law exponent. All that is needed to describe a line is two points, which means that relatively few measurements are needed to approximate the roughness power spectrum. Assuming the roughness power spectrum will be a power law, the acoustic backscatter technique can give the approximate roughness power spectrum quickly with just a few measurements.

The acoustic backscatter technique measures the 2D power spectrum of an area of the rough surface. The power spectrum of an area of rough surface with height profile $h(x,y)$ is defined as

$W(k_x, k_y) = |F[h(x,y)]|^2,$  

the magnitude squared of the Fourier transform of the height profile. The power spectrum leads to important statistical descriptions of the rough surface. From the 2D roughness power spectrum, two independent parameters describing both horizontal and vertical scales of roughness can be calculated. The rms height, which describes the variation of heights about some mean height, can be found by the equation

$h_{rms}^2 = 2 \pi \int_{k_l}^{k_H} W(k) k dk,$  

where $k$ represents the magnitude of the two-dimensional wave vector, $k$, and the factor of $2 \pi$ comes from integrating over the unit circle. Likewise, the autocorrelation function for the surface profile, $h(x,y)$, describes the spatial arrangement of roughness on the surface, and is given by

$C(x) = \frac{2 \pi}{h_{rms}^2} \int_{k_l}^{k_H} W(k) e^{-ikdk} dk.$  

The statistical values are found by integrating the roughness power spectrum over the range of roughness wave number values $k_l$ to $k_H$, the low- and high-cutoff roughness wave numbers, respectively. The correlation length, $L_c$, is defined as the length at which the correlation function drops to $1/e$ of its initial value. The correlation length value describes the spatial arrangement of roughness on the surface. A surface with roughness that changes rapidly, i.e., smaller clods, will have a shorter correlation length than a surface that has slow, undulating roughness.

The low- and high-cutoff roughness wave numbers are important to the statistical description of the roughness. The low-cutoff roughness wave number, $k_L$, is chosen based on the size of the surface being measured. If the low-cutoff roughness wave numbers were chosen corresponding to a wavelength of several kilometers, then the roughness of larger scale valleys and hills would be included in the roughness calculation. Likewise, a high-cutoff roughness wave number, $k_H$, should also be chosen. A high-cutoff roughness wave number of infinity corresponds to adding infinitesimal scaled roughness into the roughness statistics calculation. The higher the value for the high-cutoff roughness wave number that is chosen, the smaller the roughness scales that will be included in the statistical calculations. Cutoff wave numbers should be chosen with respect to the surface length being considered. For agricultural surfaces a low cutoff on
the scale of meters should be chosen. In this work, a low cutoff corresponding to 60 cm was chosen because it was the length of the transects measured by the laser profiler. The high-cutoff wave number was chosen corresponding to a wavelength of 3 mm, the approximate sampling length of the laser profiler. All roughness statistics should be referenced to the cutoff wave numbers used in calculation. In order to compare roughness parameter estimations between the laser profiler and acoustic measurements, the same cutoff wave numbers were used for both.

IV. RESULTS

A. General results and interpretation

Measurements by the laser profiler showed that the slope and intercept parameters for the power spectrum in the x- and y-directions were approximately the same for each plot. The similarity between the 1D power spectra measured for the x- and y-directions showed that the random rough surfaces were approximately isotropic. Conversion of the 1D power spectra from the x- and y-transects to 2D power spectra could occur, according to Eq. (8), since the surfaces were isotropic.

Measurements of the roughness power spectra by the laser profiler and acoustic backscatter for the soil plots coated with Saran powder dissolved in MEK are shown in Fig. 8. The error bars on the acoustic data points represent one standard deviation about the mean value of 15 acoustic measurements taken at different points on the rough soil surface. Acoustic points measured at different roughness wave number values were obtained by changing the frequency of the tone burst or by changing the graze angle.

The difference in the three soil plots can immediately be seen by viewing Fig. 8. The largest roughness plot has the greatest overall scattering power and the smaller-sized roughness plots can be seen in descending order of scattered power. Both the laser profiler and the acoustic backscatter measurements differentiate between the different-sized roughness plots. However, the acoustic technique appears to show less distinction between the two smallest roughness plots than the laser profiler technique.

The slope and intercept of the best-fit line to the three plots are listed in Table I. The intercept is defined where \( k = 1 \text{ m}^{-1} \). Since the measurements from the laser profiler spanned a larger bandwidth of values, the slope and intercept values were also deduced from different bandwidths. The laser profiler slope and intercept values were obtained over a larger bandwidth than the acoustic values. From Table I it is seen that agreement between slope values from the laser profiler and acoustic backscatter becomes better when a greater number of acoustic data points is taken. The standard errors are also listed with the values of slope and intercept for the best-fit line and show that measurements of the laser profiler and acoustic backscatter are within error of each other.

Slope and intercept values of the roughness power spectrum of the rain-sealed soil plot (plot 4) were also measured and are given in Table I. The power spectra evaluated from measurements by the laser profiler and acoustic backscatter are shown in Fig. 9. The data points represent an average of 30 measurements taken at a particular roughness wave number from different points on the surface. The error bars represent one full standard deviation about the average value of 30 measurements.

From the slope and intercept values, the rms height and correlation length are calculated over the chosen roughness wave number range. The values for the rms height and correlation length are shown in Table II. The low-cutoff wave number used in these calculations was 10 \( 1/\text{m} \), corresponding to a wavelength of approximately 60 cm, the length of the transects measured by the laser profiler. The high-cutoff wave number values were obtained by changing the frequency of the tone burst or by changing the graze angle.

<table>
<thead>
<tr>
<th>Plot</th>
<th>Slope Laser</th>
<th>Intercept Laser</th>
<th>Slope Acoustic</th>
<th>Intercept Acoustic</th>
<th>% Diff. Slope</th>
<th>% Diff. Intercept</th>
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<td>-2.10</td>
<td>±0.44</td>
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</table>
wave number was 2100 1/m, corresponding to a wavelength of approximately 3 mm, the sampling length of the laser profiler.

When the power spectrum is a power law over a range of roughness wave numbers, as is assumed for random rough surfaces, the correlation length does not depend on the intercept value but only on the slope (exponent) of the log of the power spectrum. The range of roughness wave numbers for which the power law is valid depends on the largest and smallest clod sizes existing on the rough surface. As the percent difference between the laser profiler and acoustic backscatter slope estimations increases, the percent difference between the correlation length also increases. Good estimates of the slope and intercept values are important to obtaining reliable statistical parameters describing the roughness of the surface. The slope and intercept values of the roughness power spectrum are determined by fitting a line to a series of data points. The fewer the data points, the greater the chance for error. The error inherent in any one measurement has more effect on the approximation when the number of data points is few as opposed to many. Fewer acoustic measurements were taken on the first two plots than on the last two plots. Comparing the values of the slope and intercept with the statistical values of rms height and correlation length shows that better results were obtained by taking more acoustic measurements. The results show that many acoustic data points (greater than 6) should be taken to map out an adequate portion of the roughness power spectrum, or else the estimations of correlation length for the rough surface will not be valid.

### B. The effects of roughness wave number cutoff choice

A further consideration of the statistical characterization of the roughness from porous soil surfaces is the effect of choosing a cutoff roughness wave number. The low-cutoff roughness wave number is chosen to limit the largest roughness scales that will be included in the statistical calculations. The high-cutoff roughness wave number is chosen to limit the smallest scales of roughness that will be included in the statistical estimations. Jackson et al. have previously used a high-cutoff roughness wave number of infinity for measurements of the roughness statistics for underwater surfaces. Likewise, their criterion for choosing the low-cutoff roughness wave number is that it be on the order of the lowest acoustic wave number used. In Jackson et al., the high-cutoff roughness wave number is specific and the low-cutoff roughness wave number is somewhat arbitrary and limited by the extent of the acoustic measurement.

An important question to examine is whether extrapolating the approximate power spectra beyond the roughness wave numbers measured is feasible or if calculations from the power spectra should be limited to measured values. The power-law assumption will hold over a particular range of frequencies up to a certain maximum clod size (low-roughness cutoff wave number). An immediate consequence is that extrapolating the data above the range where the power-law assumption holds would give inaccurate estimates of roughness. The power spectra are approximated by a finite number of data points over a specific range of roughness wave numbers from both the laser profiler measurements and acoustic backscatter measurements. The following paragraphs will examine the choice of different cutoff roughness wave numbers, both high and low, on the calculation of the surface roughness statistics, and the assumption that the power spectra are power law. First, the rms height and correlation length are calculated from the approximated power spectra for various roughness wave number ranges for both the laser profiler data and the acoustic backscatter data of plot 4 and compared. The first set of calculations is made to determine the magnitude of errors introduced by choosing various low- and high-cutoff roughness wave numbers. Second, the power spectra from plot 4 are approximated by limiting the analysis data over the acoustic measurements and the laser profiler measurements. By examining the roughness statistics from power spectra approximated over different wave number bandwidths, the assumption of a power-law form for the power spectrum is tested.

Examination of the effects of the choice of cutoff roughness wave numbers to the estimation of the rms height value is straightforward. Solving Eq. (15) for the case of a power law:

\[ W(k) = \beta k^{-\gamma}, \]

(17)
yields

![Power spectrum comparison](Image)

**FIG. 9.** Power spectra of rough surface sealed and weathered by rainfall as measured by the laser profiler and acoustic backscatter.
Since the exponent variable, $\gamma$, is constrained between 2 and 4, the rms height is proportional to the inverse of the low-and high-cutoff roughness wave number values raised to some positive power. Changes in the high-cutoff roughness wave number have little to negligible effect on the value of the rms height as long as the high-cutoff value is large enough. Figure 10 shows the rms height calculation, as the high-cutoff roughness wave number is increased (low cutoff remains 10 l/m) for the slope and intercept values obtained from plot 4. The high-cutoff roughness wave numbers graphed start from 130 l/m (the limit of roughness wave numbers measured by the acoustic technique) and extrapolate beyond the high-roughness wave number measured by the laser profiler (2100 l/m). As the high-cutoff wave number value increases to infinity, the rms height asymptotically approaches a certain value, around 8 mm for plot 4. In Eq. (18) it can also be seen that in the limit of $k_H$ goes to infinity, the $k_H$ term vanishes from the calculation. It is always better to have a specific cutoff value as opposed to an arbitrary value. Since the value of the rms height does not appear to change much as the high-cutoff roughness wave number becomes large and the value asymptotically approaches a specific value, the statistical measurement can be standardized by assuming the value for the high cutoff to be infinity.

Changes in the low-cutoff roughness wave number will have a more dramatic effect on the rms height value. Decreasing the low-cutoff wave number value will increase the value of the rms height while increasing the value of the low cutoff will decrease the value of the rms height. Decreasing the low-cutoff roughness wave number is the same as including larger scales of roughness in the statistical calculation. Figure 11 shows the effect of decreasing the low-cutoff roughness wave number (high cutoff remains 2100 l/m). The smaller the low-cutoff value is, the larger the rms height value estimation and the larger the percent difference between values obtained by the acoustic backscatter method and the laser profiler method. Choosing the low-cutoff roughness wave number based on the lowest wave number values measured by the laser profiler and acoustic backscatter techniques (about 10 l/m and 30 l/m, respectively) shows an increase of 20% in the rms height. As the low-cutoff roughness wave number decreases the rms height does not asymptotically approach a particular value. No absolute low-cutoff value can be chosen, but rather the low-cutoff roughness wave number should be chosen based on the lowest roughness wave number obtained by the measurement technique. If the power-law assumption is seen to break down above the lowest roughness wave number measured, a cutoff should be chosen before the power-law assumption broke down.

Similar effects for the choice of high- and low-roughness wave number cutoff can be seen for the calculation of the correlation length of the rough surface. Figure 12 shows a graph of the correlation length versus the changing high-roughness wave number cutoff (low cutoff remains 10 l/m). The percent differences between the laser profiler and acoustic backscatter values are also shown. As the high-cutoff roughness wave number is increased, smaller scales of roughness are included in the overall statistical calculation. By lining up the rough surface profile with itself, the inclusion of smaller scales of roughness means that the surface would become more uncorrelated at a shorter distance. Figure 12 shows the effect of decreasing the correlation length by including smaller scales of roughness. The absolute differences between the acoustic backscatter and laser profiler values do not change much by increasing the high-roughness wave number cutoff. However, since the overall correlation length becomes less with the inclusion of smaller scales of roughness, the relative percent difference increases significantly. As the high-cutoff roughness wave number is increased to infinity, the correlation lengths appear to asympt-
includes larger scales of roughness with itself is to increase the distance at which the profile becomes uncorrelated with itself. As the low-cutoff roughness wave number gets smaller, the correlation length becomes large. The absolute error between estimates made by the laser profiler and the acoustic backscatter technique increases dramatically, as does the relative percent difference. In going from a low-cutoff value of 30 l/m to 10 l/m, the correlation length increases by almost 150%. The dramatic increase in correlation length value with decreasing low-cutoff choice means that the choice of low-cutoff value should not extrapolate far beyond the actual measured roughness wave numbers.

The effects of the choice of low- and high-cutoff roughness wave numbers appear to follow a general trend for the case examined. Table III shows the values of rms height and correlation length for plot 4 using specific values for the low- and high-cutoff roughness wave numbers. Values for slope and intercept were also obtained for the laser profiler data of plot 4 using only the roughness wave number bandwidth measured by the acoustic backscatter technique (30 l/m to

![FIG. 12. (a) Correlation length values obtained from slope and intercept values measured for the fourth rough surface plot versus different values of high wave number cutoff (—acoustic backscatter; 1—laser profiler). (b) Percent difference between the correlation length values obtained by the laser profiler and acoustic backscatter technique.](image1)

![FIG. 13. (a) Correlation length values obtained from slope and intercept values measured for the fourth rough surface plot versus different values of low wave number cutoff (—acoustic backscatter; 1—laser profiler). (b) Percent difference between the correlation length values obtained by the laser profiler and acoustic backscatter technique.](image2)

<table>
<thead>
<tr>
<th>Cutoff wave number</th>
<th>rms height</th>
<th>% difference</th>
<th>Corr. length</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Acoustic technique</td>
<td>lo—10 l/m</td>
<td>6.88</td>
<td>(1) &amp; (2) = 1</td>
<td>2.9</td>
</tr>
<tr>
<td>(2) Laser profiler (BW—A)</td>
<td></td>
<td>6.95</td>
<td>(1) &amp; (3) = 2.5</td>
<td>3.29</td>
</tr>
<tr>
<td>(3) Laser profiler (BW—L)</td>
<td>hi—2100 l/m</td>
<td>7.06</td>
<td>(2) &amp; (3) = 1.6</td>
<td>2.6</td>
</tr>
<tr>
<td>(1) Acoustic technique</td>
<td>lo—10 l/m</td>
<td>8.23</td>
<td>(1) &amp; (2) = 5.5</td>
<td>1.89</td>
</tr>
<tr>
<td>(2) Laser profiler (BW—A)</td>
<td></td>
<td>7.8</td>
<td>(1) &amp; (3) = 3</td>
<td>2.4</td>
</tr>
<tr>
<td>(3) Laser profiler (BW—L)</td>
<td>hi—∞</td>
<td>8.48</td>
<td>(2) &amp; (3) = 8</td>
<td>1.36</td>
</tr>
<tr>
<td>(1) Acoustic technique</td>
<td>lo—30 l/m</td>
<td>3.97</td>
<td>(1) &amp; (2) = 1</td>
<td>3.93</td>
</tr>
<tr>
<td>(2) Laser profiler (BW—A)</td>
<td></td>
<td>3.93</td>
<td>(1) &amp; (3) = 3.7</td>
<td>3.93</td>
</tr>
<tr>
<td>(3) Laser profiler (BW—L)</td>
<td>hi—130 l/m</td>
<td>3.83</td>
<td>(2) &amp; (3) = 2.6</td>
<td>3.93</td>
</tr>
<tr>
<td>(1) Acoustic technique</td>
<td>lo—30 l/m</td>
<td>7.2</td>
<td>(1) &amp; (2) = 9</td>
<td>0.65</td>
</tr>
<tr>
<td>(2) Laser profiler (BW—A)</td>
<td></td>
<td>6.6</td>
<td>(1) &amp; (3) = 5.3</td>
<td>0.83</td>
</tr>
<tr>
<td>(3) Laser profiler (BW—L)</td>
<td>hi—∞</td>
<td>7.6</td>
<td>(2) &amp; (3) = 13</td>
<td>0.47</td>
</tr>
</tbody>
</table>

aBW—A= values obtained from slope and intercept estimated over acoustic measurement bandwidth.
bBW—L= values obtained from slope and intercept estimated over laser profiler measurement bandwidth.
130 l/m). Using the acoustic backscatter bandwidth (BW—A) of roughness wave numbers yielded a slope and intercept for the laser profiler power spectrum of \(-2.3 \pm 0.11\) and \(-52.4 \pm 2.15\), respectively. Table III shows that the rms height and correlation length values obtained from the laser profiler power spectrum (BW—A) do not give better agreement to the acoustic measurements than the values obtained using the power spectrum approximation from the full laser bandwidth (BW—L). If the power spectrum deviates from the power-law form outside of a particular bandwidth, comparing results obtained from two different analysis bandwidths should yield significantly different results. The fact that the roughness statistics from the laser profiler data is in good agreement between the narrowly defined bandwidth (BW—A) and the larger bandwidth (BW—L) show the power law is a good approximation to the power spectrum for the rough surface.

A comparison of correlation length values using different low-and high-cutoff roughness wave numbers corresponding to the laser profiler sampling and the wave number ranges corresponding to the acoustic measurements seems to indicate the latter gives better estimates. However, comparisons of the table values with Fig. 12 show that on average the absolute difference remains the same. Overall, choosing cutoff ranges as sampled by the laser profiler and the acoustic technique does not yield better agreement. The percent difference between the laser profiler and acoustic backscatter technique for the rms height does not change much for choice of high-cutoff value of infinity. The correlation length has a larger percent difference for high cutoff of infinity but a small absolute difference. Extrapolating the wave number range to infinity for the high-cutoff value is acceptable since it removes the arbitrariness of the choice without sacrificing a proper description of the surface by the roughness statistics.

When the cutoff wave numbers are chosen such that they are an extrapolation far beyond the maximum length of the sampled area (decreased low-cutoff value), the reliability of the estimation decreases. The percent difference between the values estimated by the acoustic backscatter technique and the laser profiler technique increases dramatically as the low-cutoff roughness wave number is decreased. Minor differences in the calculated best-fit slope and intercept propagate into large errors at the lower roughness wave numbers. Special care needs to be taken when choosing the low-cutoff roughness wave number for estimating the roughness statistics. In order to decrease the error introduced into the estimation of roughness statistics for a surface, the low-cutoff roughness wave number should not be chosen far beyond the lowest roughness wave number actually sampled by the measurement technique.

The construction of the rough surfaces explains the need to limit the low-cutoff value to the measured ranges. The rough surface plots were constructed by breaking the soil into large clods. These larger clods were further broken down into smaller and smaller clods, depending on the roughness sizes desired for a particular plot. It has been noted by Kolmogorov that the breaking of particles into smaller and smaller particles yields a fractal-like structure.\(^{36}\) In the case of a rough surface this means similar roughness on all scales. The power-law form of the power spectrum describes the fractal behavior of the rough surface. Only the smallest-sized clods that are broken off from larger clods limit the smallest scales of roughness (high-roughness wave numbers). The largest clods initially used to construct the rough surface limit the largest scales of roughness (low-roughness wave numbers). Therefore, it is reasonable to assume that the power-law structure of the rough surface would extend to infinitely high-roughness wave numbers but would not extend to infinitesimally small-roughness wave numbers. The roughness power spectrum will begin to deviate from the power-law form as the roughness wave number is sampled below the maximum clod size. The low-cutoff roughness wave number should be chosen at the point of deviation if sampled by the measurement. If the deviation from the power law is not seen in the roughness wave numbers sampled, the lowest sampled wave number should be chosen for the cutoff. Any expansion below the minimum sampled roughness wave number may give erroneous statistical results since the power-law form of the roughness power spectrum may not hold far below the sampled roughness wave numbers.

V. SUMMARY AND CONCLUSION

Theory that was previously developed to measure the roughness of porous soil surfaces from acoustic backscatter was tested through experiments.\(^1\) Four soil plots were constructed by simple farming implements. The first three plots were constructed with varying degrees of roughness and sealed with Saran powder dissolved in MEK. The fourth plot was constructed and left out in the elements (rain) for weathering. The four plots were created to mimic the kind of soil surfaces one might expect to find on real agricultural lands.

The rms height and the correlation length of rough soil surfaces were estimated by measuring the roughness power spectra of the surfaces through acoustic backscatter. The rms height describes roughness on vertical scales and the correlation length describes the spatial arrangement of the roughness on the surface. The results of the acoustic backscatter technique were then compared with measurements made by a laser profiler. Good agreement was obtained between the acoustic backscatter technique and the laser profiler technique. Both techniques were able to distinguish between surfaces with different-sized roughness.

Calculation of the roughness statistics, rms height, and correlation length is done by integrating the roughness power spectrum, approximated as a power law, over a range of roughness wave numbers. The high-cutoff roughness wave number describes the smallest sampling length used in the estimation of the roughness statistics. The low-cutoff roughness wave number corresponds to the overall length of the plot used in the statistical calculation. Good agreement was found between the laser profiler and acoustic backscatter when the low-cutoff roughness wave number was chosen to correspond to the length of 60 cm and the high-cutoff roughness wave number was chosen to correspond to a sampling length of 3 mm. The laser profiler measured the surface pro-
file in transects of length 60 cm with an average sampling distance of about 3 mm between measurements.

The roughness power spectra were calculated from the profile transects measured by the laser profiler. Different points on the roughness power spectra were also mapped out by the acoustic backscatter technique for each surface. From these data values, the roughness power spectra were approximated as a power law for both the acoustic technique and the laser profiler technique. Approximating the roughness power spectra as a power law enabled the values of the roughness power spectra to be extrapolated beyond the measured values. Examination of the effects of including different roughness wave number ranges on the roughness statistics was made by choosing different low- and high-cutoff roughness wave number values. The percent difference between the laser profiler and acoustic backscatter values for rms height and correlation length increased as the low- and high-cutoff roughness wave numbers were expanded. A dramatic increase in the percent difference in values for rms height and correlation length was seen as the low-cutoff roughness wave number was decreased. In making estimates of roughness statistics for a surface it is important to choose the appropriate roughness scales to incorporate in the final calculation. Especially important is not to extrapolate the low-cutoff roughness wave number far below the actual sampled values. Using a low-cutoff roughness wave number corresponding to a length of 3 mm when the maximum length sampled was only 50 cm may lead to large errors in the estimation of the roughness statistics. In most cases, a high-cutoff roughness wave number of infinity (no high-cutoff value) can be chosen since the roughness statistics approaches a specific value as the high-cutoff value approaches infinity. Choosing the high-cutoff value to be infinity gives uniqueness to the statistics (the asymptotic value at infinity) and provides a standard for the statistical description. Examination of the low- and high-cutoff roughness wave numbers values shows the importance of describing the surface with roughness statistics referenced to the incorporated wave number ranges.

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