Time-Domain Solution of the Temperature Increase Induced by Diagnostic Ultrasound

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Abstract – In this paper, we present a complete time-domain solution to estimate the temperature increase induced by pulsed ultrasonic fields, such as those used in diagnostic applications. Our computational model includes three steps: (1) calculation of the acoustic field, (2) of the rate of heat generation and (3) of the temperature distribution. For step 1 and 3, the acoustic and thermal fields are computed by integrating the known acoustic and thermal Green functions of the homogenous medium, respectively, over the surface of the transducer and over the volume of interest. For step 2, we derive a new expression for the instantaneous rate of heat generation. Previous expressions gave only the average rate of heat for a single frequency excitation. We finally present computational results of the temperature increase induced by a point source and by a circular focused transducer.

INTRODUCTION

For several decades, ultrasound has been used as a routine diagnostic and imaging modality in various clinical fields, especially in obstetrics. Although its use has been generally considered as safe, there are still substantial concerns about possible bioeffects as some of the newer diagnostic techniques employ higher intensities. One of the well-known effects of ultrasound is heating (or hyperthermia) caused by ultrasonic absorption in tissues. Indeed, whole-body hyperthermia is a proven teratogen in animal experiments. On the other hand, an overly conservative limitation of the output intensity of ultrasound devices may increase the risk of missing valuable diagnostic information. Therefore, there is a significant need for a reliable methodology to estimate the temperature increase induced by diagnostic ultrasound, so that its risk-free status can be maintained.

As a direct measurement of the temperature distribution inside the human body is technically difficult and possibly unsafe, computational models appear to be a reasonable solution for assessing the safety of diagnostic ultrasound. Thermal modeling is based on the Bio-Heat Transfer (BHT) equation [Pennes, 1948]. This equation relates the temperature increase to the average rate of heat $\langle q_v \rangle$ (or the heat source function), which for a single-frequency wave is given by [Nyborg, 1981]:

$$\langle q_v \rangle = \frac{\alpha P_0}{\rho_0 c_0} \tag{1}$$

where α is the frequency-dependent absorption coefficient, P_0 is pressure magnitude of the ultrasonic wave, ρ_0 and c_0 are the density of the medium and the wave speed.

The previous models that have been developed to compute the temperature increase generally assume a CW excitation signal [Ellis and O'Brien, 1996]. This condition is relevant to hyperhermia and ablation, but is not relevant to diagnostic applications where the excitation signal is a train of short, wide-band pulses. In particular, Eq.1 holds for a single frequency only and cannot be applied when the excitation signal has a significant bandwidth. To our knowledge, there has been no specific approach that would be fully relevant to diagnostic ultrasound. In this paper, we present a complete time-domain solution for estimating the temperature increase induced by a pulsed ultrasonic wave in a homogenous medium with linear absorption and blood perfusion. This solution includes a generalization of Eq.1 to the time domain.

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COMPUTATIONAL MODEL

As shown in Fig.1, our model includes three steps: (1) calculation of the acoustic pressure field, (2) of the rate of heat and (3) of the temperature distribution.

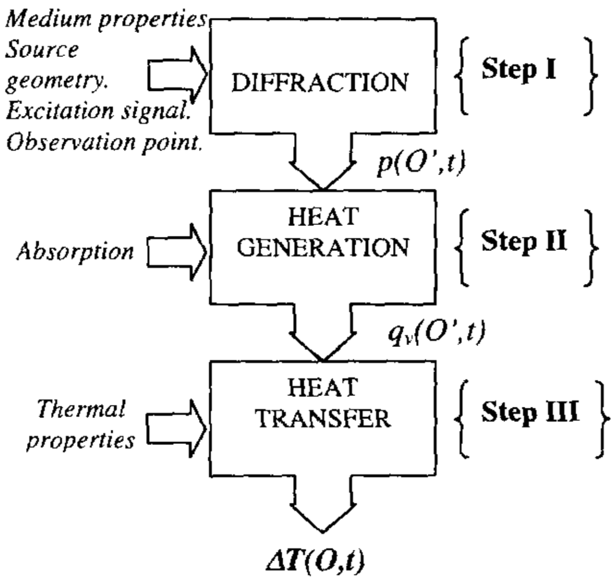

Figure 1: 3-step model to calculate the temperature increase.

Computation of the acoustic and thermal fields

Assuming that the transducer is set in a infinite, rigid baffle, the pressure pulse response at observation point O and time t is given by:

$$h_p(O,t) = \frac{\rho_0}{2\pi}\iint_{(S_0)} \gamma_n(M)\left\{a(t,r) \underset{t}{*} \frac{\delta(t-r/c_0)}{r}\right\} dS(M) \quad (2)$$

where $\gamma_n(M)$ is the acceleration distribution at the surface of the transducer, S_0 is the surface of the transducer and $a(t,r)$ is the time-domain attenuation function, as expressed by Jensen et *al.* [1993]. For an arbitrary excitation signal $s(t)$, the pressure response is simply:

$$p(O,t) = h_p(O,t) \underset{t}{*} s(t) \quad (3)$$

Assuming an unbounded medium for the thermal problem, the function of Green of the BHT equation is [Morse and Feshbach, 1953]:

$$h_T(O,O',t) = \frac{dv}{c_v}\frac{1}{(4\pi\kappa t)^{3/2}} e^{-r'^2/4\kappa t} e^{-t/\tau} \quad (4)$$

where κ is the thermal diffusivity, τ the perfusion time constant, c_v the heat capacity at constant volume and $r' = OO'$. Function $h_T(O,O',t)$ represents the temperature increase induced at point O by a point source of volume dv located at point O'. If we consider now an arbitrary heat source function $q_v(O',t)$, the temperature increase induced by heated point O' at point O is:

$$\delta T(O,O',t) = q_v(O',t) \underset{t}{*} h_T(O,O',t) \quad (5)$$

The global temperature increase is obtained by integrating Eq.5 over a volume of interest V_0:

$$T(O,t) = \iiint_{(V_0)} \delta T(O,O',t)\, dV(O') \quad (6)$$

Time-domain computation of the rate of heat

We start with a generalized equation of state that relates the acoustic pressure $p(O,t)$ to the dilatation $\theta(O,t)$ of the medium through compressibility function $c(t)$:

$$\theta(O,t) = \nabla\vec{l}(O,t) = c(t) \underset{t}{*} p(O,t) \quad (7)$$

Next, we assume an absorption law that is linear for $\omega << \omega_h$ and reaches a plateau for $\omega >> \omega_h$:

$$\alpha(\omega) = \frac{\alpha_0}{2\pi}\omega_h\left[1-\exp(-\omega/\omega_h)\right] \quad (8)$$

This allows us to derive a causal, non-singular expression for the compressibility function:

$$c(t) = c_0 - \frac{\delta(t)}{\rho_0 c_0^{\,2}} - \frac{2\alpha_0\omega_h}{\rho_0 c_0 \pi^2}\left\{\frac{\pi}{2} - \tan^{-1}(\omega_h t)\right\} \quad (9)$$

The rate of heat is derived from the Energy Balance Equation:

$$\nabla\vec{I}(O,t) + \frac{\partial w(O,t)}{\partial t} = -q_v(O,t) \quad (10)$$

where $\vec{I}$ is the intensity vector and w the disturbance energy, i.e. the sum of the kinetic and potential energies. From Eq.7, 9 and 10, we finally obtain a time-domain expression for the heat source function:

$$q_v(O,t) = \frac{\alpha_0 \omega_h}{\rho_0 c_0 \pi} p^2(O,t) - \frac{2\alpha_0}{\rho_0 c_0 \pi^2} p(O,t) \int_{-\infty}^{t} \frac{p(O,\tau)}{(t-\tau)^2 + 1/\omega_h^2} d\tau \qquad (11)$$

We can easily check that for a steady, single-frequency pressure waveform $p(t)=P_0 \sin \omega t$, the average $<q_v>$ of Eq.11 winds up to the formula in Eq.1.

RESULTS

We have applied the formulation established in the previous section (Eq.3, 6 and 11) to compute the temperature increase induced by a point source and by a focused circular transducer. Those sources radiate into a medium defined by the following properties: ρ_0=1000kg/m^3, c_0=1500m/s, α_0=2dB/cm/MHz, κ=1.5e-7m^2/s and c_v=3.8e6J/m^3.°C. Unless mentioned otherwise, the perfusion time constant is τ=980s corresponding to a perfusion length of 1.2cm. In order to compute the temperature increase at an arbitrary point O in the medium, the volume of interest V_0 around point O is divided up into elementary volumes dV_i. Each elementary volume corresponds to a heated point source O_i'. The spacing between these points O_i' is chosen to be $\lambda_{min}/4$, where λ_{min} is the smallest acoustic wavelength in the medium. We first compute the ultrasonic field at points O_i'. For the point source study, the radiated pressure response $p(O_i',t)$ is simply given by the function of Green of the acoustic problem with rigid boundaries:

$$p(O'_i, t) = P_0 \frac{\delta(t - r/c_0)}{r} \qquad (12)$$

where r is the distance from the point source to O_i'. For the focused circular transducer study, the pressure distribution is given by Eq.2 and 3 and is computed by the Discrete Representation (DR) method [Piwakowski and Delannoy, 1989]. The acceleration distribution at the surface of the circular transducer is assumed to be uniform, i.e. $\gamma_n(M) = \gamma_0$. The values of P_0 in Eq.12 and γ_0 are such that the CW output power of the source in a lossless medium is 100mW.

Figure 2, 3 and 4 illustrate the thermal response from a point source. In Fig.2, the excitation signal (section *a*) is a train of sinusoids with a central frequency of 1MHz and a repetition frequency of 100kHz. Section *b* shows the very beginning of the temperature increase up to 0.5ms at 3cm from the source. In Fig. 3 and 4, we have studied the influence of the duty cycle of the excitation signal and of the perfusion length $L = \sqrt{\kappa \tau}$ of the tissue. In both figures, the temperature increases until heat generation is perfectly compensated by heat dissipation through diffusion and perfusion. As expected, the steady value of the temperature increase is proportional to the duty cycle and to the perfusion length.

Figure 5 shows the steady-state temperature increase along the axis of a circular focused transducer. The diameter of the transducer is 2cm and its radius of curvature 1.4cm. The excitation signal is a sinusoidal pulse train with a duty cycle of 0.9, i.e. a quasi CW excitation. The computed temperature distribution is in good agreement with that obtained from a frequency-domain approach [Ellis and O'Brien, 1996].

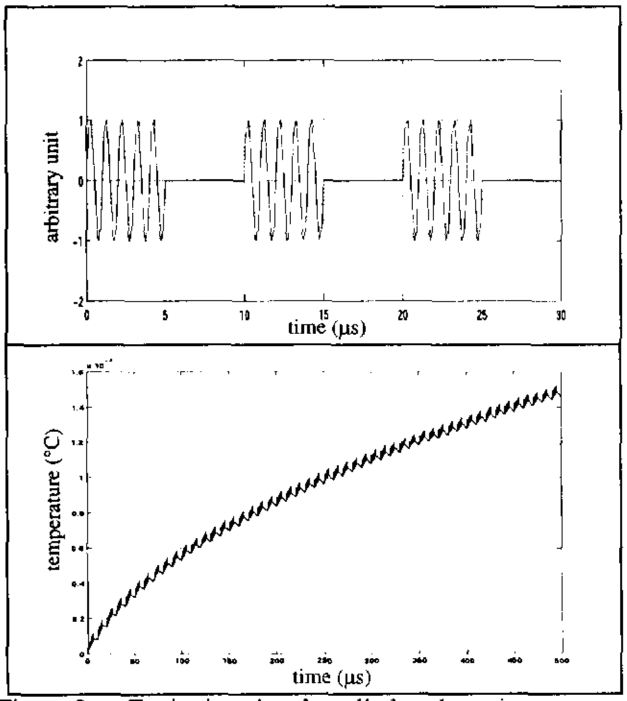

Figure 2: *a*. Excitation signal applied to the point source. *b*. Temperature increase at 3cm from the source.

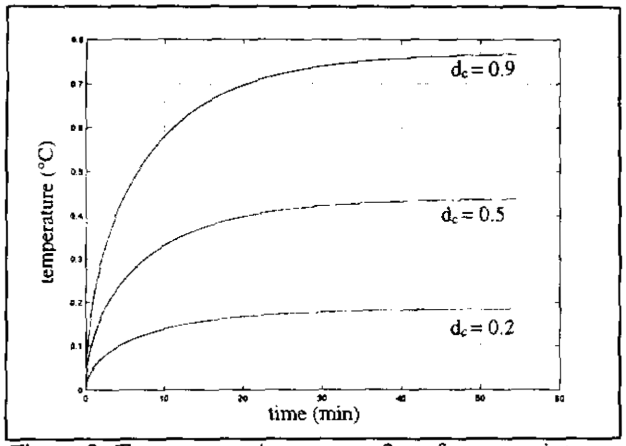

Figure 3: Temperature increase at 3cm from a point source vs. duty cycle (d_c) of the excitation signal.

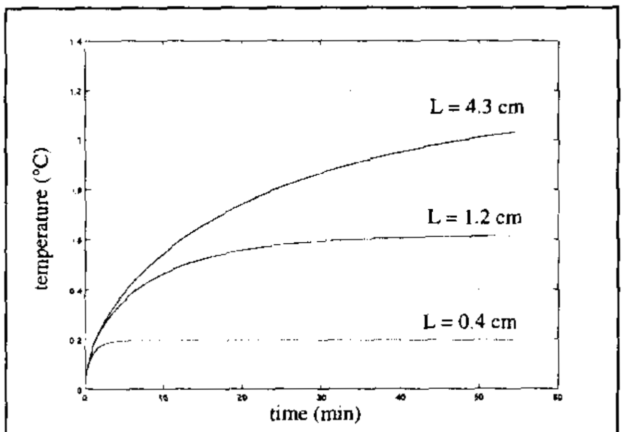

Figure 4: Temperature increase at 3cm from a point source vs. perfusion length (L) of the tissue.

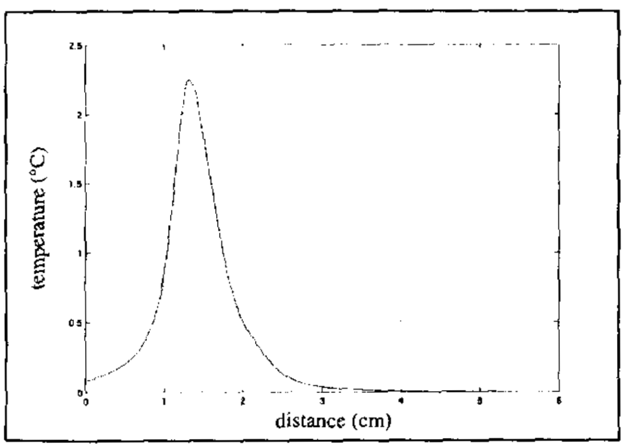

Figure 5: Steady-state temperature increase along the axis of a circular focused transducer – excitation signal is a sinusoidal pulse train with 0.9 duty cycle.

CONCLUSIONS

A computational model was developed to estimate the temperature increase induced by diagnostic ultrasound in homogenous tissues. Whereas previous studies assumed CW or quasi-CW ultrasonic signals and were not fully relevant to diagnostic applications, our model assumes wide-band signals and enables to compute all the relevant quantities (pressure, heat and temperature) in the time domain. This model was applied to compute the temperature increase induced by a point source and by a circular focused transducer excited by a train of pulses. The effect of varying the duty cycle of the excitation and the blood perfusion length was studied. The computed results displayed the expected behavior.

ACKNOWLEDGMENTS

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