INTRODUCTION TO ULTRASOUND

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I. WHAT IS SOUND?

Sound is the rapid motion of molecules. These molecular vibrations transport energy from a transmitter, a sound source like our voice, to a receiver like our ear. Sound travels in waves that transport energy from one location to another. When the molecules get closer together, this is called compression, and when they separate, this is called rarefaction. This mechanical motion, the rapid back and forth motion, is the basis for calling sound a mechanical wave or a mechanically propagated wave. Sound requires a medium in order to propagate. There are three types of medium: gas, liquid and solid. In a vacuum (such as outer space) sound cannot propagate - there is no medium.

This contribution is an introduction to the physical considerations of medical ultrasound and includes (1) a discussion of matter to understand issues of ultrasonic propagation in tissue and a means to quantify properties of matter, (2) a discussion of wave propagation and a means to quantify the temporal, amplitude, transmission and reflection characteristics of ultrasound waves and (3) concludes with a discussion of resolution trade-off issues.

II. TYPES OF ACOUSTIC WAVES

The classification of sound waves is based on the type of motion that is induced in the medium by the propagating sound wave. For purposes of ultrasonic physics, the lowest level of organization within material is called a particle (Kinsler et al., 1982). The particle is represented in the Fig 1 as dots and can be thought of as a volume of material. Each of these dots consists of millions of molecules and yet each has dimensions of a fraction of an ultrasonic wavelength. The assumptions of the particle are

\[ a \ll (\Delta V)^{1/3} \ll \lambda \]  \hspace{1cm} (1)

where (1) the typical separation between molecules, \( a \), is very small compared to the typical length of the particle, \( (\Delta V)^{1/3} \), and (2) the typical length of the particle, is very small compared to the typical separation
between maxima and minima of stresses, \( \mathcal{L} \), in the media. The former assumption assures that the forces experienced by the particle are an average over a large number of molecules and the latter assumption assures that such forces experienced by the particle are uniform.

![Longitudinal Wave Representation](image1)

![Shear Wave Representation](image2)

![Sine Wave Representation](image3)

Figure 1

When an ultrasonic wave is propagated within material, the type of wave is classified in terms of (1) the direction the ultrasonic energy is traveling and (2) the direction the particle is moving. A longitudinal wave occurs when the particles move back and forth (that is, left to right and back horizontally) relative to the direction of the wave energy, as demonstrated in Fig 1a. Propagated longitudinal waves travel through all kinds of materials: gases, liquids and solids.

In the case of shear waves, the particles move at right angles to the direction of the wave propagation as shown in Fig 1b. In this figure, the particles are moving vertically up and down while the wave energy is moving horizontally. Shear waves exist only in solid materials, not in liquids or gases, nor do they exist in soft tissues because soft tissues are approximated as a liquid. Shear waves do, however, travel in harder biological materials such as bone.

III. COMPOSITION OF MATTER

Since acoustic waves requires a material medium, an understanding of the physical properties of matter provides a basis for studying how ultrasound propagates. Physical properties of matter include: volume (it occupies space), mass (a quantitative measure of matter), weight (the way we measure mass on earth) and inertia (any body resists change in motion).

Tissue is matter. Since matter is very complex, simple models are used for illustrative purposes, namely, the three states of gas, liquid and solid. There are many physical properties which are similar with gases and liquids and because of these similarities, both are referred to as fluids.

Matter is composed of molecules. These molecules are held together by forces which, for modeling purposes, can be thought of as tiny springs or rubber bands. This model can be used to describe what happens when a sound wave moves through matter. In Fig 1, the particles are interconnected by the (invisible) springs. When the sound wave, in this case the driving force, interacts with the first group of particles, the interaction causes the particles to be pushed towards the adjacent particles. Through this process, the sound wave sets up a chain reaction, but each subsequent particle moves a little less than its neighbor due to the fact that there is friction in the system. If the sound wave, which is the driving force, changes direction, then the particles also change the direction of their movement. This movement occurs over a very short distance of several micrometers or less.

Considering only the longitudinal wave, the alternate compression and decompression action shown in Fig 1a corresponds to the crest (positive deflection) and trough (negative deflection) of the sine wave form shown in Fig 1c. Note that the location where the sine wave is a maximum corresponds to where the particles are very close together; and where the sine wave is a minimum the particles are spread apart. The correspondence between the sine wave and the particle density is important since it is far more common to diagram this with a sine wave.

Later, the ideas of material stress and strain will be discussed (Feynman, et al., 1965). Such media, whether gases, liquids or solids, are termed elastic media. An elastic medium is homogeneous when its physical properties are not dependent upon the location in the medium. Consider, for example, that the molecular composition and density of a very small volume element within a material are measured at many different locations. If the composition and density are the same at all points, the material is said to be homogeneous. Otherwise, the material is said to be inhomogeneous.

An elastic medium is isotropic when its physical properties do not influence differently the direction that a wave may travel through it. If the physical properties which affect the transport of energy (such as heat, electricity, light, sound) are the same in all directions, the material is said to
be isotropic. Otherwise, the material is said to be anisotropic.

IV. DENSITY, ELASTICITY AND SPEED

Density is a property common to all matter, but also is a property that makes different types of matter unique (Feynman, et al., 1965). It is an important material property which has a direct affect upon ultrasonic properties such as speed. Density is defined in two ways. The one with which we deal in ultrasonics is called mass density, and is defined as,

\[ \rho = \frac{m}{V} \]  

(2)

where \( m \) is the mass and has the unit of kilogram (kg) and \( V \) is the volume and has the unit of meter cubed (m\(^3\)). The other, weight density, is defined in terms of the object’s weight divided by its volume.

If one tries to change the size or shape of a solid by applying a force, the object will resist the attempt by trying to return to its initial condition once the deforming force is removed. That is, if the solid is deformed, it will return to its original shape and size when the cause of the deformation is removed. Elasticity is the property of recovering size and shape when the forces producing deformations are removed.

Elasticity is quantified by relating force to deformation and this ratio of “force” to “deformation” is called an elastic modulus (Feynman, et al., 1965). The deforming force, called stress, is represented as force per unit area and has the unit of newton per meter squared (N/m\(^2\)) or pascal (Pa):

\[ \text{STRESS} = \frac{F}{A} \]  

(3)

The deformation, called strain, is represented in terms of a relative change in dimensions when subjected to a stress, that is,

\[ \text{STRAIN} = \frac{\Delta L}{L} \]  

(4)

The ratio of stress to strain is called the elastic modulus

\[ \text{ELASTIC MODULUS} = \frac{\text{STRESS}}{\text{STRAIN}} \]  

(5)

All substances exhibit the property of elasticity and hence each can be quantitatively described in terms of an elastic modulus. There are a number of different types of elastic moduli, each based on how the force is applied to the object. The more common types of elastic moduli include: Young’s modulus, shear modulus and bulk modulus. Bulk modulus is one of the elastic moduli that is most often associated with fluid media, that is, liquids and gases (Kinsler et al., 1982). The reciprocal of bulk modulus is called compressibility and, thus, has the unit of reciprocal pascals (Pa\(^{-1}\)). It is more common to refer to fluids, which are quite compressible, in terms of their compressibility instead of their elastic modulus.

The speed of sound depends on both the density and elasticity of materials, that is (Kinsler et al., 1982),

\[ c = \sqrt{\frac{\text{ELASTIC MODULUS}}{\text{DENSITY}}} \]  

(6)

In terms of the three physical states of matter (gas, liquid, solid), Table 1 quantifies and demonstrates how elasticity and density affect speed.

| Typical elasticity and density values for the three states of matter. |
|--------------------|----------------|----------------|
| TYPICAL ELASTICITY | TYPICAL DENSITY | CALCULATED SPEED |
| (Pa)               | (kg/m\(^3\))  | (m/s)          |
| GAS                | \(10^5\)       | 316            |
| LIQUID             | \(10^9\)       | 1000           |
| SOLID              | \(10^{11}\)    | 4472           |

V. TISSUE AS MATTER

From an acoustics point of view, the physical properties of tissue can be classified as a quasi-liquid or a quasi-solid (Dunn and O’Brien, 1976). The prefix quasi means “seeming like a” and is appropriate because the
acoustical behavior of tissue sometimes behaves like a liquid and sometimes like a solid. To the touch, tissue appears to be a solid and yet some of its acoustical properties are very similar to those of water, a liquid.

The physical properties of tissue are influenced by and composed of water, ions, macromolecules and cells and are a consequence of the chemical structures of fibrous and nonfibrous components. Tissues are divided into various kinds, including epithelial, muscular, connective, nervous, blood, etc. Each of these has different physical properties. Common to all tissues is a large amount of water. Selected physical properties of pure water at 37°C (98.6°F) are listed in Table 2 (Nyborg, 1975). The physical properties of tissue depend strongly upon water because water makes up almost three-quarters of the entire mass of the human body. The water concentration varies from tissue to tissue with vitreous humor quite high at around 99%, liver at 70%, skin at 60%, cartilage at 30% and adipose as low as 10%.

**TABLE 2**

Selected physical properties of pure water at 37°C.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressibility</td>
<td>$4.4 \times 10^{-10} \text{ Pa}^{-1}$</td>
</tr>
<tr>
<td>Bulk modulus</td>
<td>$2.3 \times 10^9 \text{ Pa}$</td>
</tr>
<tr>
<td>Density</td>
<td>$990 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Speed</td>
<td>$1527 \text{ m/s}$</td>
</tr>
</tbody>
</table>

Ions affect the physical properties of tissue. When sodium chloride (NaCl), an electrolyte, is added to water, the density of the solution increases and the compressibility decreases (elasticity increases). The addition of electrolytes causes the water molecules to form a hydration sheath of relatively high density and high elasticity around each ion. At physiological concentrations, the density is about 0.6% greater and the elasticity is about 2% greater than for distilled water. Since elasticity increases at a greater rate than density, from the discussion in Section IV, it can be seen that speed increases.

Collagen is an important macromolecule which influences the physical properties of tissue. Collagen is a high tensile strength, insoluble fiber found in most connective tissues, such as the connective tissues of cartilage, tendon, bone, skin and muscles. It is the most abundant protein in the human body. It constitutes twenty-five to thirty-three percent of the total protein, and therefore, about six percent of the total body weight. Collagenous fibers exhibit an elastic modulus approximately 1000 times greater than that of other tissues.

Fat is an almost water free tissue. Total body water is dependent upon the total amount of body fat. At least ten percent of the body weight is comprised of lipid (fat), the most abundant of which are the triglycerides, which are found throughout the body, as well as in certain specialized connective tissues, namely the adipose tissue.

**VI. FREQUENCY, WAVELENGTH AND SPEED**

We have many perceptions of the nature of sound. The idea of pitch refers to our perception of frequency, that is, the number of times a second that air vibrates in producing sound that we hear. Voices are classified according to pitch in which the lowest frequency is a bass voice and the highest frequency is a soprano voice. This description of frequency, however, is limited to the frequency range, or spectrum, over which humans can hear sounds. There are sound frequencies below and above what humans can hear. The acoustic spectrum is shown in Fig 2a.

![Acoustic Spectrum](image)

The lowest frequency classification in the acoustic spectrum is
Ultrasound

infrasound which has a frequency range below 20 Hz. Audible sound is what we hear and has an approximate frequency range of 20 Hz to 20 kHz. The ultrasound frequency range starts at a frequency of 20 kHz. Examples of devices that emit frequencies at the lower frequency end of the ultrasonic spectrum are a dog whistle and industrial ultrasonic cleaners.

Medical ultrasound equipment operates in the ultrasonic frequency range between 1 and 15 MHz (Fig 2b). Therapeutic (physical therapy) applications operate around 1 MHz. For most diagnostic applications in abdominal and OB-GYN ultrasound and in echocardiography, the frequency range is between 2.25 and 7.5 MHz. For very superficial body parts, such as the thyroid and the eye, and peripheral vascular applications where ultrasound does not have to penetrate very deeply into the body, higher ultrasonic frequencies in the range of 7.5 to 15 MHz can be used because ultrasonic attenuation increases with increasing frequency.

Ultrasound travels in waves that emanate from a source. The high crests and low troughs represent specific amplitude values of the wave and correspond to peak compressional and peak rarefactive values. The distance from one crest to the next, or from one trough to the next, has a particular distance associated with it and is called the wavelength and denoted by \( \lambda \) in Fig 3a.

\[
\begin{align*}
\text{(a) Amplitude vs Distance} & & \text{(b) Amplitude vs Time} \\
\end{align*}
\]

\[c = \frac{\lambda}{T} = \lambda f\]  

Figure 3

The time that it takes for one cycle to occur is called the period (Fig 3b). The period (T) is the reciprocal of frequency (f), that is,

\[
f = \frac{1}{T}
\]

In the diagnostic ultrasound frequency range, for example, the period for a frequency of 5 MHz is 0.2 \( \mu \)s (200 ns).

As demonstrated in Fig 3, the horizontal axis can illustrate either distance (Fig 3a) or time (Fig 3b). This is an important concept in diagnostic ultrasonic instrumentation. Distance information can be converted to time values, and time converted to distance information. Ultrasonic instruments are constantly performing these conversions in order to display sonographic images. The space (or distance) over which one cycle travels is called the wavelength and the time which one cycle occupies is called the period, that is, wavelength is "distance/cycle" and period is "time/cycle." Speed is the constant that relates wavelength (\( \lambda \)) to period:

\[
c = \frac{\lambda}{T} = \lambda f
\]

where, for medical applications, the tissue's propagation speed, \( c \), is assumed to be constant at 1540 m/s. In the diagnostic ultrasound frequency range, for a frequency of 3.5 MHz, the wavelength is 0.44 mm (440 \( \mu \)m).

Propagation speed is very important in the proper design of diagnostic ultrasound systems. It must be known in order for the instrumentation to convert time values into distance or depth information, because the diagnostic system keeps track of only time. In order for the ultrasound instrumentation to perform this function, the speed must be set at a constant value. Although different tissues have different speeds, all of the propagation speeds in soft tissues fall within a rather narrow range. This narrow range allows for the use of an average speed which causes only a small margin of error when calculating distances or ranges in the body.

The propagation speed which has been most accepted for soft tissue is 1540 meters per second (1540 m/s, or 1.54 mm/\( \mu \)s). Another way to look at propagation speed is in terms of its reciprocal speed, which is 0.649 \( \mu \)s/mm or 6.49 \( \mu \)s/cm. This means that the wave travels a distance of 1 cm every 6.49 ms. If a structure is positioned 1 cm from the ultrasound source, then it would take 6.49 \( \mu \)s for the ultrasound wave to reach that structure and an additional 6.49 \( \mu \)s for an echo from that reflecting structure to return. Thus, the round trip reciprocal speed is 12.98 \( \mu \)s/cm and, for convenience, is usually quoted as 13 \( \mu \)s/cm. For example, if an object is located 10 cm from the source, then it would take 130 \( \mu \)s for the ultrasound wave to reach the reflecting structure and return to the source.

VII. TEMPORAL CHARACTERISTICS

There are two basic generation modes of ultrasound used in medical ultrasound (Fig 4). Generation mode means the way in which the ultrasonic
wave is "shaped" when it is transmitted from the ultrasonic transducer, that is, the waveform's temporal characteristics. One way is to continuously excite the ultrasonic transducer with an electrical sine wave at a constant amplitude. This produces a continuous ultrasonic wave at the same frequency as that of the electrical frequency and is termed continuous wave ultrasound (CW mode or CW ultrasound), as shown in Fig 4a.

![Continuous Wave Representation](image)

Another way is to turn on the ultrasound for a very short period of time and turn it off for a much longer period of time and then to repeat this process. This is accomplished by exciting or shocking the ultrasonic transducer with very short electrical signals, waiting for some time and repeating the electrical shocking. The ultrasonic waves that are generated are termed pulse wave ultrasound (PW mode or PW ultrasound), as shown in Fig 4b.

![Pulsed Wave Representation](image)

Figs 3 and 4a show a CW ultrasound wave. To describe quantitatively a CW waveform, only two quantities are required, that is, amplitude and frequency (or period - see Eq 7).

To quantify a waveform of a single pulse (one of the pulses shown in Fig 4b), an additional piece of information is the time during which the pulse is on, termed the pulse duration ($\tau$). If the number of cycles per pulse is N, then the pulse duration is

$$\tau = N \cdot T$$  \hspace{1cm} (9a)

and from Eq 7

$$\tau = \frac{N}{f}$$  \hspace{1cm} (9b)

To quantify a waveform of repeated pulses (Fig 4b), in addition to amplitude, frequency and pulse duration, the rate at which pulses are repeated is required, and quantified by either the pulse repetition frequency (PRF) or its reciprocal, the pulse repetition period (PRP).

The ratio of the pulse duration to the pulse repetition period is called the duty factor (DF), that is,

$$DF = \frac{\tau}{PRP} = \tau \cdot PRF$$  \hspace{1cm} (10)

For example, if the pulse duration is 1 $\mu$s and the pulse repetition period is 1 ms (PRF = 1 kHz), then the duty factor is 0.001.

To summarize, the four quantities required to quantify a repeated pulse are: amplitude of the pulse, frequency of the ultrasonic signal in the pulse, pulse duration and pulse repetition frequency. Although these four quantities are sufficient, other combinations of these four quantities can also be used to quantify a repeated pulse waveform.

VII. WAVEFORM QUANTITIES IN A MEDIUM

A necessary concept to understand axial (or range) resolution is the distance one cycle (and hence one pulse) occupies in a medium. The distance one cycle occupies in a medium is the wavelength, $\lambda$ (Eq 8).

For a repeated pulse waveform, as shown in Fig 5, the distance one pulse occupies in a medium is called the spatial pulse length (SPL), that is, the number of wavelengths per pulse where

$$SPL = N \cdot \lambda$$  \hspace{1cm} (11a)

and from Eqs 9a and 7, respectively,

$$SPL = \frac{\tau}{T} \cdot \lambda = \tau \cdot f \cdot \lambda$$  \hspace{1cm} (11b)
For example, at an ultrasonic frequency of 7.5 MHz, the wavelength in tissue is 0.205 mm and, for a three cycle pulse, the spatial pulse length is 0.615 mm.

**Figure 5**

**IX. AMPLITUDE CHARACTERISTICS**

In this section, amplitude quantities will be derived from basic principles under conditions in which the medium is assumed lossless and infinite in extent. It is the solution to the one-dimensional wave equation that yields the quantitative relations between the ultrasonic amplitude quantities (Kinsler et al., 1982). The variables required to develop the lossless, one-dimension, acoustic wave equation are displacement, $\xi(x, t)$, density, $\rho(x, t)$ and pressure, $p(x,t)$ of a particle (Eq 1). Acoustic wave propagation, and the development of its wave equation, can be approached from the Equation of State which describes the change in density to the change in pressure, the Continuity Equation which relates particle motion to the change in density by invoking the conservation of mass principle and the Equation of Motion which compares the change in pressure to particle motion through Newton's Second Law of Dynamics.

The Equation of State is

$$p = \rho_c \left( \frac{dp}{d\rho} \right) \text{ at constant } \rho_c$$  \hspace{1cm} (12)

where the propagation speed $c_0$ is the derivative term

$c_0^2 = \left( \frac{dp}{d\rho} \right) \text{ at constant } \rho_c$ \hspace{1cm} (13)

and, where the development was done under the small-signal conditions,

$$p = P_0 + p \quad p \ll P_0$$ \hspace{1cm} (14a)

$$\rho = \rho_0 + \rho_c \quad \rho_c \ll \rho_0$$ \hspace{1cm} (14b)

where $P_0$ and $\rho_0$ are the ambient pressure and undisturbed density, respectively, and $p$ and $\rho_c$ are the acoustic pressure and excess density, respectively. For a perfect gas, under adiabatic conditions where there is no heat transfer, that is, $pV^\gamma = $ constant, where $\gamma$ is the ratio of specific heats, Eq 13 yields

$$c_0^2 = \frac{\gamma P_c}{\rho_0}$$ \hspace{1cm} (15)

The Continuity Equation expresses mass conservation as

$$\rho_c = -\rho_0 \frac{\partial p}{\partial x}$$ \hspace{1cm} (16)

which, in turn, expresses the fraction change in position of the particle displacement to the fraction change in density as

$$s = -\frac{\partial \xi}{\partial x} = \frac{\rho - \rho_0}{\rho_0} = \frac{\rho_c}{\rho_0}$$ \hspace{1cm} (17)

where $s$ is called condensation.

The Equation of Motion is expressed as

$$\frac{\partial p}{\partial (x+\xi)} = -\rho \frac{\partial^2 \xi}{\partial t^2}$$ \hspace{1cm} (18)

Combining Eqs 12, 16 and 18 and linearizing yields the wave equation

$$\frac{\partial^2 \xi}{\partial t^2} = c_0^2 \frac{\partial^2 \xi}{\partial x^2}$$ \hspace{1cm} (19)

The solution to Eq 19 considers one-dimensional traveling wave
characteristics of the particle displacement, $\xi(x, t)$, particle velocity $u(x, t)$, particle acceleration $a(x, t)$ and acoustic pressure $p(x, t)$ in the positive $x$ direction in a lossless medium, infinite in extent for a single frequency function. Assuming a solution to the wave equation as

$$\xi(x, t) = \xi_0 \cos (\omega t - kx)$$ (20)

where $\xi_0$ is the amplitude particle displacement, $\omega = 2\pi f$ and $k$ is the wave number ($= \omega/c_0$) and from this assumed solution, the particle displacement and particle acceleration is obtained, where

$$u(x, t) = \frac{\partial \xi}{\partial t} = - U_0 \sin (\omega t - kx)$$ (21a)

and

$$a(x, t) = \frac{\partial u}{\partial t} = - A_0 \cos (\omega t - kx)$$ (21b)

where $U_0$ and $A_0$ are their respective amplitude terms. To determine the acoustic pressure, $p(x, t)$, Eqs 12, 13 and 16 are combine to yield

$$p(x, t) = \rho_0 c_0^2 \frac{\partial \xi}{\partial x}$$ (22)

thus yielding

$$p(x, t) = - \rho_0 \sin (\omega t - kx)$$ (23)

In summary, the relationships between these amplitude terms are

$$\xi_0 = \frac{U_0}{\omega} = \frac{A_0}{\omega^2} = \frac{p_0}{\omega \rho c_0}$$ (24a)

$$U_0 = \omega \xi_0 = \frac{A_0}{\omega} = \frac{p_0}{\rho c_0}$$ (24b)

$$A_0 = \omega^2 \xi_0 = \omega U_0 = \frac{\omega p_0}{\rho c_0}$$ (24c)

$$p_0 = \rho_0 c_0 \xi_0 = \rho_0 c_0 U_0 = \frac{\rho_0 c_0 A_0}{\omega}$$ (24d)

For example, in water at 20°C where $\rho_0 = 998$ kg/m$^3$ and $c_0 = 1481$ m/s, at an ultrasonic frequency of 1 MHz, if $\xi_0 = 18.5$ nm (185 Å), then $U_0 = 12$ cm/s, $A_0 = 7.3 \times 10^{-4}$ m/s$^2$ and $p_0 = 180$ kPa (1.8 atm).

X. WAVE PROPAGATION PROPERTIES

The propagation properties generally used to describe quantitatively the propagation of ultrasound in materials are speed, impedance and attenuation. The propagation of ultrasound is assumed to be an adiabatic process, that is, a process in which heat conduction does not occur. Therefore, the speed at which ultrasonic energy propagates in an isotropic fluid is (Pierce, 1981; Kinsler et al., 1982; Hall, 1987)

$$c_0 = \sqrt{\frac{B_{AD}}{\rho_0}}$$ (25)

where $B_{AD}$ is the adiabatic bulk modulus. Compared to Eq 6, the elastic modulus for an isotropic fluid is $B_{AD}$. As such, ultrasonic waves propagated in fluids are longitudinal waves. In a gas,

$$B_{AD} = \gamma p_0$$ (26a)

and

$$c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$ (26b)

which agrees with Eq 15.

For a liquid, the elastic modulus is

$$B_{AD} = \gamma B_T$$ (27a)

where $B_T$ is the isothermal bulk modulus and, therefore,

$$c_0 = \sqrt{\frac{\gamma B_T}{\rho_0}}$$ (27b)

In an isotropic solid, both longitudinal and shear waves are supported wherein their respective propagation speeds are

$$c_L = \sqrt{\frac{Y(1-\sigma)}{\rho_0(1+\sigma)(1-2\sigma)}}$$ (28a)

and

$$c_S = \sqrt{\frac{Y}{2\rho_0(1+\sigma)}}$$ (28b)

where $Y$ is the Young's modulus and $\sigma$ is the Poisson's ratio. Since $\sigma$ is
less than 0.5, c_L is greater than c_s.

The specific acoustic impedance of the wave is defined as the ratio of the acoustic pressure to particle velocity. For plane waves under the conditions of Eqs 21a and 23, the specific acoustic impedance is defined as

\[ Z_s = \frac{p(x, t)}{u(x, t)} \]  \hspace{1cm} (29a)

where

\[ Z_s = \frac{-p_0 \sin(\omega t - kx)}{u_0 \sin(\omega t - kx)} = \rho_0 c_0 \]  \hspace{1cm} (29b)

For other than plane waves, \( Z_s \) is generally different, that is, \( Z_s \) depends upon both the medium and the wave type (plane, cylindrical, spherical, etc). The \( \rho_0 c_0 \) product is encountered frequently in analytic acoustics and is called the characteristic acoustic impedance of the medium or simply the acoustic impedance. Only for a plane wave are these two impedances the same. For an isotropic fluid, combining Eqs 29 with Eq 25 yield the characteristic acoustic impedance as

\[ Z = \rho_0 c_0 = \sqrt{\rho_0 c_0 B_{AD}} \]  \hspace{1cm} (30)

and is a property only of the medium. The unit of the acoustic impedance is the rayl (kg/m²s), after Lord Rayleigh. Table 3 summarizes the numerical ranges of \( \rho_0, c_0 \) and \( Z \) for the various isotropic media.

| TABLE 3 |
|-----------------|-----------------|------------------|
| **Typical density, propagation speed and characteristic acoustic impedance values for isotropic media.** | | |
| \( \rho_0 \) (kg/m³) | \( c_0 \) (m/s) | \( Z \) (rayl) |
| GAS | 1 | 100-1000 | 100-1000 |
| LIQUID | 1000 | 1000-2000 | 1-2 x 10⁶ |
| SOLID | 1000-10,000 | 2000-10,000(L) | 10-100 x 10⁶(L) |
| | | 1500-5000(S) | 4-50 x 10⁶(S) |

where L represents a longitudinal wave and S a shear wave.

XI. TRANSMISSION AND REFLECTION PHENOMENA

When an ultrasonic traveling wave impinges upon an acoustically discontinuous boundary, part of the energy is transmitted across the boundary into the second medium and part is reflected (Kinsler et al., 1982; Ensminger, 1988), as shown in Fig 6. If the incident wave in fluid medium 1 (in which the characteristic acoustic impedance \( Z_1 = \rho_1 c_1 \)) impinges at an angle \( \theta_i \) relative to the normal of the boundary surface, then the reflected wave's angle is \( \theta_i \) and the transmitted wave's angle is \( \theta_t \) because of Snell's Law

\[ k_1 \sin \theta_i = k_2 \sin \theta_t = k_2 \sin \theta_i \]  \hspace{1cm} (31)

where \( k_1 (= \omega/c_1) \) and \( k_2 (= \omega/c_2) \) are the wave numbers in the two media, and \( \theta_i, \theta_t, \text{ and } \theta_r \) are, respectively, the incident, reflected and transmitted angles relative to the normal of the boundary surface. Therefore, \( \theta_i \) and \( \theta_t \) are both equal to \( \theta_i \) because they are propagating in the same media with the same propagation speed, \( c_1 \), and \( \theta_r \) is the same as \( \theta_2 \). Eq 31 is more traditionally written as

\[ \frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2} \]  \hspace{1cm} (32)

There are three conditions to evaluate Eq 32, that is, (1) normal incidence when \( \theta_1 = 0 \), (2) oblique incidence when \( c_1 \) is greater than \( c_2 \), and (3) oblique incidence when \( c_1 \) is less than \( c_2 \).
For normal incidence, \( \theta_1 = 0 \), and from Eq 32, \( \theta_2 = 0 \). For oblique incidence, if \( c_1 > c_2 \), then \( \theta_1 > \theta_2 \) which means the transmitted beam is bent towards the normal of the boundary surface as is shown in Fig 6.

For oblique incidence, if \( c_1 < c_2 \), then \( \theta_1 < \theta_2 \) which means the transmitted beam is bent away from the normal of the boundary surface. For this condition, a special case arises when the transmitted angle \( \theta_2 \) is 90°. This condition is referred to as the critical angle for the incident angle, \( \theta_1 \), that is,

\[
\theta_c = \sin^{-1} \left( \frac{c_1}{c_2} \right)
\]  
(33)

To evaluate the fractional amount of energy that is reflected from and transmitted across the boundary, as shown in Fig 6, two general quantities are used, one in term of the sound power reflected or transmitted and the other in terms of the sound intensity reflected or transmitted. The Sound Power Reflection (\( \alpha_r \)) and Transmission Coefficients (\( \alpha_t \)) are defined as, respectively,

\[
\alpha_r = \frac{\text{Reflected Sound Power}}{\text{Incident Sound Power}} = \frac{W_r}{W_i}
\]  
(34a)

and

\[
\alpha_t = \frac{\text{Transmitted Sound Power}}{\text{Incident Sound Power}} = \frac{W_t}{W_i}
\]  
(34b)

and the Sound Intensity Reflection (\( \beta_r \)) and Transmission Coefficients (\( \beta_t \)) are defined as, respectively,

\[
\beta_r = \frac{\text{Reflected Sound Intensity}}{\text{Incident Sound Intensity}} = \frac{I_r}{I_i}
\]  
(35a)

and

\[
\beta_t = \frac{\text{Transmitted Intensity Power}}{\text{Incident Sound Intensity}} = \frac{I_t}{I_i}
\]  
(35b)

Because of conservation of energy,

\[
\alpha_r + \alpha_t = 1
\]  
(36)

but the sum of \( \beta_r \) and \( \beta_t \) is not necessary unity.

In general, for oblique incidence,

\[
\alpha_t = \frac{(Z_2 \cos \theta_1 - Z_1 \cos \theta_2)^2}{(Z_2 \cos \theta_1 + Z_1 \cos \theta_2)^2}
\]  
(37a)

and

\[
\alpha_t = \frac{4 Z_1 Z_2 \cos \theta_1 \cos \theta_2}{(Z_2 \cos \theta_1 + Z_1 \cos \theta_2)^2}
\]  
(37b)

and

\[
\beta_t = \alpha_t = \frac{(Z_2 \cos \theta_1 - Z_1 \cos \theta_2)^2}{(Z_2 \cos \theta_1 + Z_1 \cos \theta_2)^2}
\]  
(37c)

and

\[
\beta_t = \frac{4 Z_1 Z_2 \cos^2 \theta_1}{(Z_2 \cos \theta_1 + Z_1 \cos \theta_2)^2}
\]  
(37d)

Note that \( \alpha_r = \beta_t \) because the beam cross-sectional areas of the incident and reflected areas are equal whereas \( \alpha_t \neq \beta_t \) because the beam cross-sectional areas of the incident and transmitted areas are, in general, not equal, except under normal incidence conditions.

At normal incidence where \( \theta_1 = 0 \), Eqs 37 become

\[
\alpha_t = \beta_t = \frac{(Z_2 - Z_1)^2}{Z_2 + Z_1}
\]  
(38a)

and

\[
\alpha_t = \beta_t = \frac{4 Z_1 Z_2}{(Z_2 + Z_1)^2}
\]  
(38b)

Two cases under normal incidence conditions of particular interest for medical ultrasound are (1) the acoustic impedances of the two media are very different and (2) the acoustic impedances are very similar.

For case (1) where either \( Z_1 \gg Z_2 \) or \( Z_2 \gg Z_1 \), Eqs 38 become

\[
\alpha_r = \beta_r = 1
\]  
(39a)

and

\[
\alpha_t = \beta_t = 0
\]  
(39b)

which means that virtually all of the incident energy is reflected and almost none is transmitted.

For case (2) where \( Z_1 = Z_2 \), Eqs 38 become
\[ \alpha_i = \beta_i = 0 \] (40a)

and
\[ \alpha_i = \beta_i = 1 \] (40b)

which means that virtually all of the incident energy is transmitted and almost none is reflected.

For medical ultrasound applications, it is very common that the characteristic acoustic impedances between two adjacent tissue types are within 1% or less of each other. For the case where the difference is 1%, that is, \( Z_1 = 1.500 \) Mrayles and \( Z_2 = 1.515 \) Mrayles, at normal incidence,
\[ \alpha_r = \beta_r = 0.0000248 \] (41a)

and
\[ \alpha_i = \beta_i = 0.9999752 \] (41b)

which means that only 0.00248% of the power (or energy) is reflected back from the boundary and 99.99752% is transmitted into the next medium. The reflected signal is about 46 dB down relative to the incident signal and for diagnostic ultrasound applications, tissue reflection coefficients range as low as 75 dB.

\section*{XII. Resolution Trade-Offs and Concepts}

The classical engineering trade-off of diagnostic ultrasound instrumentation is that between resolution and the depth of the image (depth of penetration). Both are directly affected by the ultrasonic frequency. As frequency is increased, resolution improves and penetration decreases. Resolution improves because the ultrasonic wavelength in tissue decreases (becomes a smaller number). Wavelength is inversely related to frequency; increase one and the other decreases.

As frequency increases, the ultrasonic attenuation also increases. Penetration is directly affected by the tissue attenuation coefficient which, in turn, is directly related to frequency. At an ultrasonic frequency of 1 MHz, an "average" attenuation coefficient for soft tissue is approximately 0.7 dB/cm whereas at 2 MHz, it is 1.4 dB/cm. Thus, the attenuation coefficient is directly related to frequency; increase one and the other increases. Thus, the attenuation coefficient can be normalized to frequency as 0.7 dB/cm-MHz.

Resolution is the ability to image or resolve discrete structures. Resolution is determined by many components and properties of the instrumentation and patient including transducer type, beam geometry, frequency and bandwidth; receiving and processing electronics; video monitor; and tissue attenuation and sound speed. For simplicity, it is easier to understand resolution by considering two types of resolution, viz., axial resolution and lateral resolution.

\textit{Axial resolution} (also termed range resolution or depth resolution) is the ability to resolve discrete structures along the beam axis. Quantitatively, it is represented as the minimum distance between two structures at different ranges at which both can just be discretely identified as two separate structures. The best axial resolution is represented by the expression

\[ \text{best axial resolution} = \frac{\text{SPL}}{2} = \frac{N\lambda}{2} \] (42)

where SPL is the spatial pulse length (see Eq 11). The transducer design affects the minimum number of cycles. More highly damped transducers (also referred to as low Q transducers) produce very few cycles of ultrasound when excited by the pulser voltage. If \( N = 3 \), at ultrasonic frequencies of 3.5 MHz (\( \lambda = 0.44 \) mm) and 7.5 MHz (\( \lambda = 0.21 \) mm), then, from Eq 42, the best axial resolutions are 0.67 mm and 0.32 mm, respectively. As the frequency increases, and other quantities remain constant, axial resolution improves.

The term "best axial resolution" has been employed because, in practice, the receiving and processing electronics affect axial resolution as does the quality of the video monitor. The electronics and monitor are often lumped into the term "system Q." Low-valued system Qs provide better axial resolution than do high-valued ones.

\textit{Lateral resolution} is the ability to resolve discrete structures perpendicular, or lateral, to the beam axis. Quantitatively, it is represented as the minimum distance between two side-by-side structures at the same range at which both can just be discretely identified as two separate structures. The best lateral resolution is represented by the expression

\[ \text{best lateral resolution} = \text{minimum beam width}. \] (43)

The "best lateral resolution" term is employed here for the same reasons that the term "best axial resolution" was used.

Fig 7 shows the beam width of two, unfocused, plane piston source transducers (operating at the same frequency) of different radii, \( a_1 \) and \( a_2 \), where \( a_2 \) is greater than \( a_1 \). For the \( a_2 \) transducer, the distance of the near field is longer but the beam width is also wider. In other words, the best
lateral resolution of a₂ transducer is worse than that of a₁ transducer in the near field. However, in the far field, at a sufficient range, the best lateral resolution is worse for the a₁ transducer demonstrating that lateral resolution is a function of imaging depth.

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**Introduction to Ultrasound**

When an ultrasonic field is focused, the focal range occurs in the near field of the transducer. Fig 8 shows the beam width from the same transducer operating at the same frequency but for two different focal lengths (focal length is the distance along the beam axis from the transducer to the focus). For the longer focus length, the minimum beam width is greater than that for the short focus case. The best lateral beam width at focus (BW) is directly proportional to wavelength (λ) and focal length (ROC, which stands for radius of curvature) and is inversely proportional to the transducer diameter (D), that is,

\[
BW = \frac{1.4 \lambda \text{ROC}}{D}
\]

(45)

In imaging terminology, the term "f-number" or "fⁿ" is often used to quantitate focusing where the lower the f-number value, the better is the focusing. The best lateral beam width at focus is related to the fⁿ by

\[
BW = 1.4 \frac{\lambda \text{ROC}}{D} = 1.4 \lambda f^n
\]

(46)

where, at the same frequency, BW can be improved by decreasing the fⁿ (the ratio of ROC to D). Another way to improve lateral resolution at the focus is to decrease the wavelength (increase frequency).

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The range of the near field (also termed the Fresnel zone) is a function of transducer dimensions and wavelength, λ, by the expression

\[
\text{near field range} = \frac{r^2}{\lambda}
\]

(44)

where r is the transducer radius. For an unfocused, plane piston source transducer, the best lateral resolution in the near field is affected mainly by transducer radius. Wavelength also affects lateral resolution here in terms of maintaining the same lateral resolution over the near field range.

---

In summary, axial resolution is affected by the wavelength, number of cycles per pulse and system Q. As axial resolution improves, the wavelength decreases (frequency increases), the number of cycles per pulse decreases and the system Q decreases. Lateral resolution is affected by the wavelength, transducer size and geometry (focusing), and focal range. As lateral resolution improves, the wavelength decreases (frequency increases), transducer size increases and the focal range decreases. Away from the focal range axially, the lateral resolution quickly deteriorates.
XIII. REFERENCES


