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Density, Elasticity, and Speed

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The following is an excerpt from *Ultrasonic Physics and Instrumentation for Sonographers,* which is tentatively scheduled for publication in 1988 by Little, Brown and Co. The purpose of this excerpt is to provide a basic background of the physical principles underlying the concepts of *Density* and *Elasticity* and their relationship to *Propagation Speed.* In addition to worked-out examples, multiple-choice problems are provided.

DENSITY

Density is a property that is common to all matter, but also is a property that makes different matter unique. It is an important material property that has a direct effect upon the ultrasonic properties such as speed. Density is defined in two ways. The one with which we deal in ultrasonics is called mass density and is defined as the *mass* of the object divided by its *volume.* The other, weight density, is defined in terms of the object's *weight* divided by its *volume.* Recall that the units of mass and weight are usually kilogram (kg) and pound (lb), respectively.

Throughout this discussion, the term *density* will mean *mass density.* In physics, it is often necessary to describe quantitatively the mass of an object and be able to compare this mass with other objects. Therefore, in order to provide this capability for comparison, the term *density* is used and means the *mass per unit volume.* The unit of mass is a kilogram, and that of volume is cubic meter (m^3). The Greek letter *rho* (ρ) is traditionally used to denote density, which, in mathematical form, is

$$\rho = m/V \tag{1}$$

where m is mass and V is volume. Therefore, the unit of density is kilograms per cubic meter. In these units, the density of a gas is about 1, that of a liquid is about 1000, and for a solid, about 1000 to 10,000.

Specific gravity of a substance is commonly measured to provide clinical information. Specific gravity is a normalized (sometimes called a relative) density, in that it is defined as the density of the material divided by the density of water, thus being unitless. It is not used in ultrasonic physics, but is

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Example 1

Consider three materials: (1) air, (2) water, and (3) steel. Suppose that you have a cube of each. The dimensions of the cube are 10 cm × 10 cm × 10 cm. The mass of these three materials is 1 g, 998 g, and 7.8 kg, respectively.

(A) What is the volume? Represent answer in units of cm^3 and m^3.

(B) What are the densities (Equation 1) of air, water, and steel? Represent answer in units of kg/m^3.

Answers:

A: 1000 cm^3 or 0.001 m^3 or 10^{-3} m^3

B: 1 kg/m^3, 998 kg/m^3, 7800 kg/m^3

simply intended to provide an example of how density is used in daily situations. Objects with a density less than water will float, like ice. Likewise, if the object's density is greater than water, it will sink, like steel.

ELASTICITY

If one tries to change the size or shape of a solid by applying a force, the object will resist the attempt by trying to return to its initial condition once the deforming force is removed. That is, if the solid is deformed, it will return to its original shape and size when the cause of the deformation is removed. The property of recovering size and shape when the forces producing deformations are removed is called elasticity, and its elasticity is a measure of how well the solid returns to its original shape. This property of matter is exhibited by all gases, liquids, and solids, although it is most commonly thought of with respect to solids.

This elastic property of materials is related to the electrical forces or bonds that hold the atoms and molecules together. Consider, for example, a rubber band. By stretching it, you have applied a deforming force. Releasing the rubber band removes the force, and the rubber band returns to its original size and shape.

Elasticity is quantified by relating force to deformation, and the ratio of force to deformation is called an elastic modulus. Perhaps one of the most familiar laws of elastic bodies is credited to Robert Hooke (1635–1703), an English experimental scientist, for the law that states that the degree an elastic body bends or stretches out of shape (is deformed) is in direct proportion to the force acting on it. The deforming force, called stress, is represented in terms of a force per unit area and has units of Newton per square meter, or Pascals:

$$\text{Stress} = \text{Force/Area} \qquad (2)$$

Consider a metal rod (like a short piece of a coat hanger) that has a diameter of 2 mm. Its cross-sectional area is (remember that diameter is twice the radius)

$$\begin{aligned} A_{\text{circle}} &= \pi r^2 \\ &= \pi\,(0.1\ \text{cm})^2 \\ &= 0.0314\ \text{cm}^2 \end{aligned}$$

or

$$\begin{aligned} A_{\text{circle}} &= \pi\,(0.001\ \text{m})^2 \\ &= 0.00000314\ \text{m}^2 \\ &= 3.14 \times 10^{-6}\ \text{m}^2. \end{aligned}$$

If this rod is pulled at each end with a force of 1 kN (1000 N), then the stress, as determined from Equation 2, is

$$\begin{aligned} \text{Stress} &= 1\ \text{kN}/0.0314\ \text{cm}^2 \\ &= 31847\ \text{N/cm}^2 \end{aligned}$$

or

$$\begin{aligned} \text{Stress} &= 1\ \text{kN}/0.00000314\ \text{m}^2 \\ &= 318471340\ \text{N/m}^2 \\ &= 3.18 \times 10^8\ \text{Pa} \\ &= 318\ \text{MPa} \end{aligned}$$

The deformation, called strain, is represented in terms of a relative change in dimensions, like a change in length, when subjected to a stress. Mathematically, this is represented as

$$\text{Strain} = \text{Change in Length/Original Length} \qquad (3)$$

For the metal rod, let's assume that its length is 50 cm. When the stress of 318 MPa is applied, it increases in length 500 μm. Thus, the strain from Equation 3 is

$$\begin{aligned} \text{Strain} &= 500\ \mu\text{m}/0.5\ \text{m} \\ &= 0.0005\ \text{m}/0.5\ \text{m} \\ &= 0.001 \\ &= 1 \times 10^{-3} \end{aligned}$$

In terms of units, stress can/is represented as meter per meter or simply unitless.

Thus, the ratio of stress to strain is called the elastic modulus:

$$\text{Elastic Modulus} = \text{Stress}/\text{Strain} \quad (4)$$

and the elastic modulus is in terms of the unit Newton per square meter (N/m^2). Note that this is the same unit as that of pressure and, therefore, the Pascal (Pa) is also the unit used with the elastic modulus. The elastic modulus for the steel rod is

$$\begin{aligned}\text{Elastic Modulus} &= 318\ \text{MPa}/1 \times 10^{-3} \\ &= 3.18 \times 10^{11}\ \text{Pa}\end{aligned}$$

There are a number of different types of elastic moduli, each based on how the force is applied to the object. The more common types are termed Young's modulus, shear modulus, and bulk modulus. For purposes of this text, it is sufficient to have an appreciation for the concept of elastic modulus; it is not necessary to understand the distinction between the various types.

According to this concept of elasticity, steel must be classified as more elastic than rubber, for example, because steel, even though not easily deformed, returns to its original shape more readily than does rubber.

All substances exhibit the property of elasticity, and hence each can be quantitatively described in terms of an elastic modulus. As a rule of thumb, these elastic moduli can be grouped or classified according to the state of matter: solids, 10^{11} Pa; liquids, 10^9 Pa; and gases, 10^5 Pa.

Example 2

Consider three hanging structures, a steel wire, a plastic thread, and a rubber band. Each of these hanging structures is connected to separate supports in the ceiling. Each are 1 m long and have a cross-sectional area of 0.1 cm^2. When the same load is separately hooked to the end of each hanging structure, the steel wire is stretched 1 μm, the plastic thread is stretched 0.1 mm, and the rubber band is stretched 50 cm. The load imparts a force of 2 N when hooked to one of the hanging structures.

Calculate the following:

(A) Cross-sectional area of the hanging structures in units of m^2.
(B) Stress on each hanging structure (Equation 2).
(C) Strain on each hanging structure (Equation 3).
(D) Elastic moduli of each hanging structure (Equation 4).

Answers:

A: 10^{-4} m^2
B: 200,000 N/m^2 or 2×10^5 Pa
C: 10^{-6}, 0.0001 or 10^{-4}, 0.5 (unitless)
D: 2×10^{11} Pa, 2×10^9 Pa, 4×10^4 Pa

One of the elastic moduli is most often associated with fluid media, that is, liquids and gases. This modulus term is called the bulk modulus. The reciprocal of bulk modulus is called compressibility and, thus, would have units of inverse or reciprocal Pascals, that is, Pa^{-1} or m^2/N.

$$\text{Bulk Modulus} = 1/\text{Compressibility} \quad (5)$$

It is more common to refer to fluids, which are quite compressible, in terms of their compressibility instead of their elastic modulus, although both are proper and correct.

Consider a liquid which has a bulk modulus of 0.95×10^9. Its compressibility is

$$\begin{aligned}\text{Compressibility} &= 1/0.95 \times 10^9\ \text{Pa} \\ &= 1.05 \times 10^{-9}\ \text{Pa}^{-1}\end{aligned}$$

DENSITY AND ELASTICITY CONSIDERED TOGETHER

The speed of sound depends on both the density and elasticity of materials. Speed (*c*) is a measure of how fast something moves and is mathematically defined as the distance moved (Δd) divided by the time it takes for the movement (Δt); that is,

$$c = \Delta d/\Delta t \quad (6)$$

There are a number of "rules of thumb" which state whether the speed is increasing or decreasing when these material properties increase or decrease. Let us see whether we can understand this.

The speed of sound is related to density and elasticity by the following equation:

Example 3

The following table of fluids lists values for their bulk modulus.

Mercury 0.27×10^9 Pa
Ethanol 0.91×10^9 Pa
Water 2.2×10^9 Pa
Glycerine 4.8×10^9 Pa
Air 1.01×10^5 Pa

Calculate the compressibility for each of these fluids (Equation 5).

Answers:

3.7×10^{-9} Pa^{-1}, 1.1×10^{-9} Pa^{-1}, 4.5×10^{-10} Pa^{-1}, 2.1×10^{-10} Pa^{-1}, 9.9×10^{-6} Pa^{-1}

TABLE 1. Typical Elasticity and Density Values for the Three States of Matter*

	Typical Elasticity (Pa)	Typical Density (kg/m^3)	Calculated Speed (m/sec)
Gas	10^5	1	316
Liquid	10^9	1000	1000
Solid	10^{11}	5000	4472

*Speed is calculated from these typical material properties from Equation 7.

$$\text{Speed} = \sqrt{\text{Elasticity/Density}} \quad (7)$$

This equation states that you first determine the ratio of elasticity to density and then take the square root of that ratio. For example, assume that elasticity equals 1.5×10^5 Pa and that density equals 0.8 kg/m^3. Their ratio is determined as follows:

$$\text{Elasticity/Density} = (1.5 \times 10^5 \text{ Pa})/(0.8 \text{ kg/m}^3)$$
$$= 1.875 \times 10^5 \text{ Pa m}^3\text{/kg}$$
$$= 1.875 \times 10^5 \text{ m}^2\text{/sec}^2$$

From Equation 7, speed is the square root of this ratio, that is,

$$\text{Speed} = 443 \text{ m/sec}$$

In terms of the three physical states of matter (gas, liquid, solid), Table 1 quantifies and demonstrates how elasticity and density affect speed.

Example 4

The elastic modulus (Pa) and density (kg/m^3) for the three liquids ethyl alcohol, water, and glycerine are:

	Elastic Modulus	Density
Ethyl alcohol	0.91×10^9	0.81
Water	2.04×10^9	1.00
Glycerine	4.76×10^9	1.26

Calculate the speed of sound for these three liquids (Equation 7).

Answers:

1060 m/sec, 1428 m/sec, 1944 m/sec

Now that we can see how elasticity and density affect the speed of sound, let us devise our own "rules of thumb." The first one is: *As the elasticity of a material increases, speed increases.* Another one is: *As the density increases, the speed increases.* Both of these are true and are useful to remember. The reason they are true relates to the fundamental properties of materials wherein neither elasticity nor density is independent of the other. Both elasticity and density of materials "track" one another, with both increasing or decreasing together, in general. Speed increases as the material becomes more organized (gas to liquid to solid) because the magnitude of elasticity (term in the numerator) increases faster than does density (term in the denominator).