

Interlaboratory Comparison of Ultrasonic Attenuation and Speed Measurements

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A set of test samples, all containing ultrasonically equivalent tissue-mimicking material, was produced and measurements of ultrasonic speed and ultrasonic attenuation coefficients were made at seven laboratories using various techniques. The ultrasonic speed values agree well with one another, having a spread of about 0.3 per cent; thus, speed values for tissue parenchyma appearing in the literature are likely to be accurate. Values of ultrasonic attenuation coefficients agree fairly well with one another, with differences between individual values and the group mean of generally less than 20 per cent of the group mean. (Key words: ultrasound, attenuation; attenuation coefficient; speed; velocity)

A considerable body of literature exists in which measurements of ultrasonic attenuation coefficients and speed in tissues have been reported.^{1,2} Many different measurement techniques, laboratories, and tissue types have been involved in these studies. Values of attenuation coefficients determined at different laboratories differ by as much as a factor of two for the same tissue parenchyma and the same species of animal (see, e.g., Hueter³ and

Frizzell⁴ for in vitro measurements in beef liver at 25°C). On the other hand, ultrasonic speed values for a single type of tissue parenchyma tend to agree within 1 or 2 per cent between laboratories. Thus, interest in the present study centers upon the degree of agreement between laboratories regarding measurements of attenuation coefficients.

In the present study, samples of ultrasonically tissue-mimicking (TM) material, all having the same composition, were produced at the University of Wisconsin, and measurements of ultrasonic attenuation coefficients and speed were conducted at seven laboratories in the United States with these TM samples.

Various measurement techniques were employed, particularly regarding determinations of the ultrasonic attenuation coefficients. In this article, we present a comparison of the measurements of attenuation coefficients and speed as measured at seven laboratories.

TEST SAMPLES OF TISSUE-MIMICKING MATERIAL

The tissue-mimicking (TM) material used has been described elsewhere.⁵⁻⁷ It consists of a water-based gelatin including *n*-propanol and a macroscopically uniform distribution (suspension) of microscopic graphite particles. The mass percentage of the various components in all samples

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Table 1. Mass Percentage of Constituents of the Tissue-Mimicking Material

Material	Mass Percentage
Water	77.72
<i>n</i> -Propanol	3.27
Dry gelatin	12.65
Graphite powder	5.51
40% Formalin solution	0.84

produced for this study are shown in table 1. The concentration of *n*-propanol determines the ultrasonic speed and contributes to maintaining the material bacteria-free. The concentration of graphite powder determines the slope of the frequency-dependent ultrasonic attenuation coefficient. The formaldehyde in the formalin solution produces chemical "cross-linking" of the gelatin resulting in a melting point for the TM material which exceeds 100°C; i.e., the formalin produces thermal stability. The formaldehyde presumably also contributes to freedom of the material from bacterial invasion.

The TM material was contained in five pairs of cylindrical enclosures. The containers for the TM materials had cylindrical acrylic walls of inner diameter 7.6 cm and wall thickness 6.3 mm. The ends of the cylinders were covered with 50- μ m-thick Saran Wrap (Dow Chemical Company, Midland, Mich.). Each member of the pair of cylinders was identical to the other except that one was about 5 cm long and the other about 2.5 cm long. Each particular pair was distinguished from the others by application of a labeling letter onto the acrylic walls.

METHODS OF MEASUREMENT

Each of the participants in the project was asked to contribute a brief description of their measurement techniques. As a result the content of the descriptions below varies according to the diversity of the authors' writing styles.

University of Wisconsin and SRI International

The ultrasonic attenuation coefficients and speed for all five pairs of samples were measured at 22°C at the University of Wisconsin. These measurements were performed using a narrow-band through-transmission technique.⁸ The same technique was used at SRI International for measurements on one pair at 25.5°C.

In using this technique to measure ultrasonic attenuation coefficients, determinations of receiver amplitudes in a water bath were made with and without the samples positioned in the beam. Corrections for transmission losses at the Saran Wrap

windows were accounted for by taking advantage of the availability of pairs of test cylinders which were identical except for their thicknesses. The ultrasonic attenuation coefficient, α , in decibels per centimeter at the center frequency was given by the expression

$$\alpha = \frac{20}{d_l - d_s} \log_{10} \frac{A_s/A_{os}}{A_l/A_{ol}}, \quad [1]$$

where d_l is the thickness of the longer test cylinder and d_s is that of the shorter cylinder, A_s/A_{os} is the ratio of receiver signal amplitudes with and without the shorter test sample in the beam, and A_l/A_{ol} is that ratio for the longer test sample.

Ultrasonic speed was measured by determining the shift in arrival time, Δt , of the received signal when the sample, of thickness d , was inserted into the ultrasonic beam. The speed, c , in the TM material is given by

$$c = \frac{c_w}{1 + c_w \Delta t/d}, \quad [2]$$

where c_w is the ultrasonic speed for the water in the water bath.

Yale University

Measurements at Yale University involved another form of narrow band through-transmission technique for determining the ultrasonic speed and attenuation coefficients. The sample holder was placed between a pair of source and receiver transducers in a constant temperature water bath maintained at 22°C. The transducers were air-backed 25-mm-diameter Valpey-Fisher polished 1-MHz X-cut quartz. Measurements were taken with the ultrasonic beam passing through the test sample in each direction (sample reversed for second measurement). The mean of these two values was used. The source transducer was driven by a Matec 6600 pulsed modulator and receiver. The narrow band pulses were typically 30 cycles long, reasonably characterized by a single (fundamental) frequency. The receiver transducer was connected to the input of a 7A18 amplifier in a Tektronix 7904 oscilloscope with a 7B92 dual-time base. The amplitude of the received pulse, with and without the tissue sample between the transducers, was measured and the attenuation coefficient calculated. Details of this calculation and associated corrections (exclusive of corrections for the Saran Wrap layers) have been published.⁹ Measurements were taken at 1, 3, and 5 MHz. The 5-cm sample could not be inserted between the transducer pair; therefore, measurements were made only on the 2.5-cm sample. Corrections supplied by Madsen⁷ were used to eliminate the effect of reflections at the Saran Wrap windows.

Ultrasonic speed was calculated using the arrival time change of the zero crossing of a cycle near the pulse center occurring when the sample was inserted between the source and receiver transducers as described elsewhere.⁹

University of Michigan

Ultrasonic speed measurements were performed at the University of Michigan using a pair of matched, unfocused, 3.5-MHz, 6-mm-diameter, broad-band transducers separated by 16.7 cm and mounted in a water-filled tank at 22°C. Shifts in acoustic broad-band pulse propagation times, occurring when test samples were inserted into the beam, were monitored using a digital intervalometer. Triggering occurred at the leading edge of the pulse.

Frequency-dependent, ultrasonic attenuation coefficient measurements were performed using a pair of 19-mm-diameter, confocal 3.5-MHz broad-band transducers separated by 20 cm. Acoustic rf pulses transmitted through the samples were digitized at 20 MHz and stored for eventual processing using a log spectral difference method.¹⁰ The difference in thicknesses of the two samples was used to eliminate effects of the Saran Wrap walls in the calculation.

Center for Devices and Radiological Health (CDRH)

A through-transmission technique, using broad-band plane wave pulses, was one method employed by CDRH to determine ultrasonic attenuation coefficients.¹¹ A transducer having a 6.35-cm-diameter piezoelectric disk was used to create the broad-band pulses. The pulses were transmitted through one of the TM samples at a time. A polymer hydrophone received the pulses, and the resultant signals were time-gated to exclude edge waves. The difference between the logarithmic spectra yields ultrasonic attenuation coefficients as a function of frequency for a broad range of frequencies.

A second method employed by CDRH to determine the ultrasonic attenuation coefficients is a radiation force, through transmission techniques.¹² A microbalance was used with a suspended 6-cm-diameter air-backed conical reflecting target. Single frequency air-backed lithium niobate transducers were used to provide a stable acoustic power output. Power output measurements were made with and without the test sample in the beam at discrete frequencies from approximately 1 to 5 MHz. The logarithm of the ratio of the transmitted acoustic powers for the two samples provides the data necessary for determining the ultrasonic attenuation coefficients of the TM material.

University of Illinois

The speed of sound was determined at the University of Illinois in a fashion similar to that used by the University of Wisconsin and SRI International (see above). A 5-MHz, 1-inch-diameter, focused transducer served as the source and a hydrophone probe with a 1-mm-diameter PZT 5-A ceramic disk acted as the receiver. The source was driven with a Panametrics 5050 PR pulser-receiver and the received wave form was digitized with an 8-bit 50-MHz analog-to-digital converter and stored for later analysis. The shift occurring in arrival time, Δt , of the received pulse was determined when the sample was inserted into the water path. Using this shift along with a measurement of the sample thickness, d , allowed calculation of the speed of sound via Eq. [2]. The temperature of the water and sample was maintained at 22.5°C.

The ultrasonic attenuation coefficients were determined using two different systems: 1) a broad band, spectral subtraction technique,¹³ and 2) a narrow band, phase-insensitive, radiation force technique.^{14,15} The former utilized the same apparatus as that used for the measurement of ultrasonic speed. The basic procedure used was that described for discrete frequencies by the University of Wisconsin and SRI International above, which resulted in equation [1]; in this case, however, a broad band system was employed giving data over the frequency range 3.3 to 6.3 MHz. The temperature was 22.5°C. The ultrasonic attenuation coefficients determined at various frequencies were fitted with the relation $\alpha = \alpha_0 f^n$, where α is the attenuation coefficient, f is the frequency, and α_0 and n are constants determined from the fit.

The radiation force measurements of attenuation coefficients were performed at 22°C at discrete frequencies using the 2.5-cm sample only because the 5.0-cm sample was too large for the experimental apparatus. The transducer was excited by a frequency synthesizer (Hewlett-Packard 8660D) driving a wide band amplifier (Electronic Navigation Industries, Model 310L) and the output level was controlled using a precision attenuator (Kay Elemetrics). The receiver target of absorbing SOAB rubber (B. F. Goodrich) was shielded from convection currents by an acoustic window. The force on the target was determined using a Cahn Model RG electrobalance with an attached chart recorder (Houston Instruments, Model 2000). The insertion loss, IL, was determined from the relation

$$IL = \frac{\ln(F_0/F)}{2d}, \quad [3]$$

where F and F_0 are the forces with and without the sample in the field, respectively, and d is the sample thickness. These data were converted to decibels per centimeter and corrected by Madsen⁷

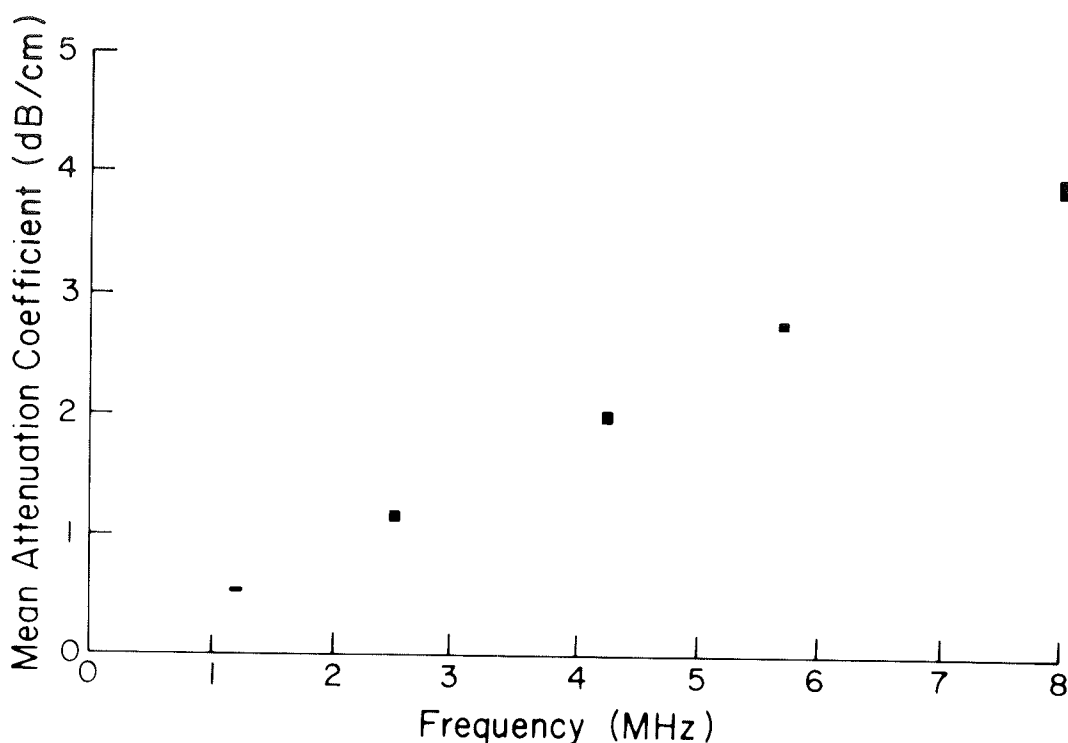


Figure 1. Average values of the ultrasonic attenuation coefficients for all five sets of samples measured at the University of Wisconsin. The upper or lower value for each rectangle corresponds to the average value before the samples were sent to the other laboratories. The other value (lower or upper) corresponds to the average of measurements after the five sets of samples had been returned. At two of the five frequencies the upper value corresponds to the earlier average.

to eliminate the effects of the two Saran Wrap interfaces.

University of Rochester

Measurements of the ultrasonic attenuation coefficients were done using the radiation force technique.¹⁶ The apparatus is very similar to that used at the University of Illinois (above), with the major difference being that microbalance (Scientek) readings were obtained from the scale of an analog microvoltmeter (Hewlett-Packard). The technique for measuring the ultrasonic speed has been reported elsewhere.¹⁶

INTERLABORATORY COORDINATION

Measurements of the ultrasonic attenuation coefficients at five frequencies between 1 and 8 MHz on all five pairs of samples were made at the University of Wisconsin about one week following the production of the test samples. Measurements of the ultrasonic speed at 2.50 MHz were also performed on all test samples at that time.

The five pairs of coded samples were then sent to the six laboratories outside of Wisconsin. One pair was sent to two different laboratories. Included with each pair was a 2.5-cm-thick test cylinder of laboratory-grade castor oil for use as a standard material if desired. Correction factors for

the Saran Wrap windows on the castor oil samples were also supplied to each laboratory.⁷ Test samples were returned to the University of Wisconsin following the measurements. The results of the measurements at the University of Wisconsin were not provided a priori to any of the other six participating laboratories.

After all five pairs of test samples had been returned to Wisconsin, measurements of the ultrasonic attenuation coefficients and speed were repeated in the identical fashion used to make mea-

Table 2. The Mean Attenuation Coefficients, $\bar{\alpha}$, for the Five Samples Measured at the University of Wisconsin and the Corresponding Standard Deviations, σ

Frequency (MHz)	$\bar{\alpha}_i$ (dB/cm)	σ_i (dB/cm)	$\bar{\alpha}_f$ (dB/cm)	σ_f (dB/cm)
1.20	0.53	0.02	0.53	0.02
2.50	1.13	0.03	1.21	0.04
4.25	1.97	0.09	2.04	0.06
5.70	2.76	0.06	2.80	0.06
8.00	4.00	0.08	3.86	0.18

Note: $\bar{\alpha}_i$ and σ_i refer to the initial values from measurements made before sending the samples to the various laboratories, and $\bar{\alpha}_f$ and σ_f refer to the final values from measurements made following the return of the samples.

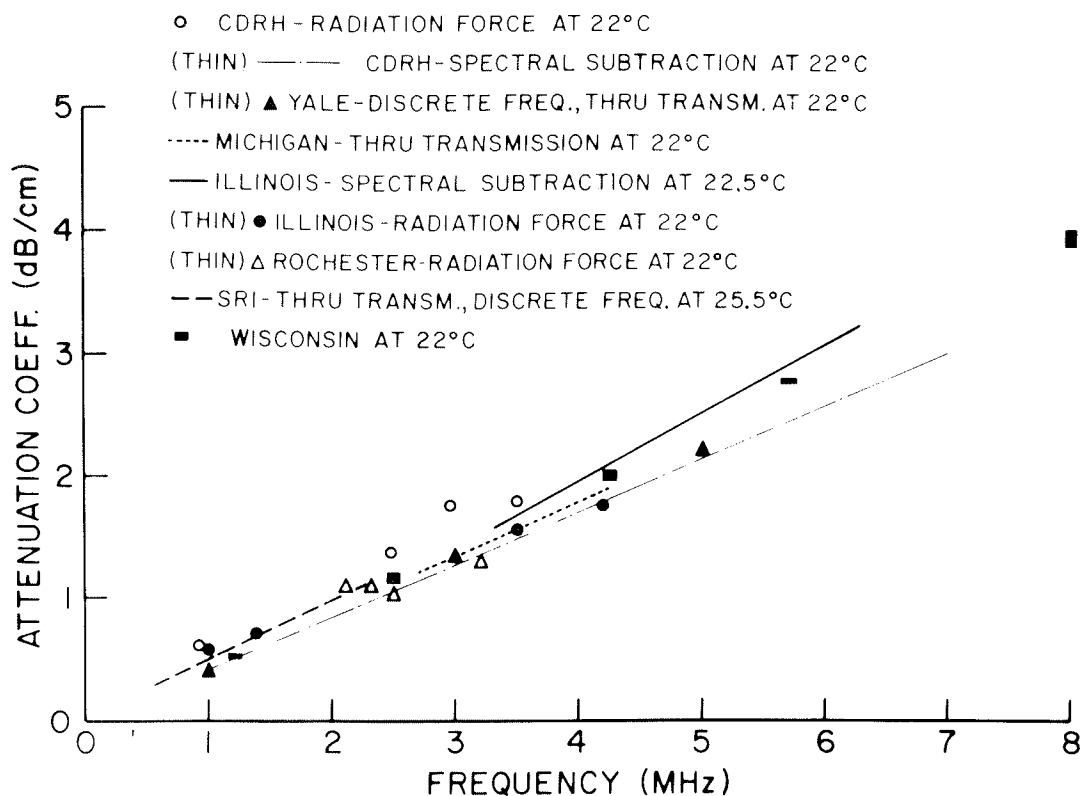


Figure 2. Values of the ultrasonic attenuation coefficients measured at all seven laboratories. The four dashed or solid lines correspond to curve-fitted data as supplied by the respective laboratories and apply only for the frequency range over which they extend. In the key, the designation (thin) means that only the thinner sample (~ 2.5 cm thick) was employed in the measurements. In the remaining, both samples were employed.

surements before the samples had been distributed.

RESULTS

Ultrasonic Attenuation Coefficients

The results of measurements of the ultrasonic attenuation coefficients for all five pairs of samples made at the University of Wisconsin at the beginning and end of the study are shown in figure 1. A rectangle is plotted at each of the five frequencies involved. The widths of these rectangles (corresponding to the direction of the frequency axis) are all the same and the value of this width has no significance. The heights of the rectangles (corresponding to the direction of the attenuation coefficient axis) do have significance, however. In particular, the upper end of each plotted rectangle corresponds to the higher of the two values of attenuation coefficients measured at that frequency and the lower end to the lower of the two values. For two of the five frequencies the upper value corresponded to the early set of measurements (made before the samples were sent out), and at the remaining frequencies the upper value corresponded to the final set of measurements (made following measurements at the six laboratories out-

side of Wisconsin). The actual mean values plotted in figure 1 are shown in table 2 along with the corresponding standard deviations. These results demonstrate that the ultrasonic attenuation properties of the samples had not changed significantly over the entire measurement period.

The values for attenuation coefficients measured in all seven laboratories are shown in figure 2. Four sets of results were reported in terms of a curve-fitted function applying over some specified frequency range. These are shown as lines extending over the four frequency ranges. The four are distinguished on the basis of whether the line is solid, dashed, or dotted. The remaining results correspond to values at discrete frequencies and are shown as circles, squares, etc. The key to the various lines and symbols representing the measurements is displayed in figure 2. In some cases only the thin sample (2.5 cm) was used and this is noted in the key as well.

As described in the *Methods of Measurement* section, two laboratories (CDRH and the University of Illinois) each employed two different methods: spectral subtraction and radiation force. Thus, four sets of data are represented in figure 2 for these two laboratories. The results from the University of Wisconsin correspond to a reproduction of figure 1 in figure 2.

Table 3. Speeds of Sound Measured in the Various Laboratories

Sample Used	Institution	Temperature (°C)	Speed of Sound (m/sec)
(All)	Univ. of Wisc. (before)	22	$\bar{c} = 1561.2 \pm 0.8$
(All)	Univ. of Wisc. (after)	22	$\bar{c} = 1559.0 \pm 0.4$
A	Yale Univ.	22	1560
B	Univ. of Mich.	22	1561
C	Univ. of Illinois	22.5	1564
D	Univ. of Rochester	23	1561
E	SRI International	25.5 (22.5)	1564 (1559)

Note: The value obtained at SRI International corresponds to a temperature of 25°C which is about 3°C higher than that at the other labs; the value in parentheses, corresponding to 22.5°C, results from application of a correction factor of 1.6 m/sec/°C to the 25°C result. The symbol \bar{c} refers to the average value for all five samples, and \bar{c} includes the standard deviation of the values averaged.

Ultrasonic Speeds

The values of ultrasonic speeds for the various laboratories are given in table 3 along with the temperatures at which the measurements were performed.

DISCUSSION

The tissue-mimicking materials used in this study exhibit ultrasonic attenuation coefficients and an ultrasonic speed representative of soft tissues. The test samples also have well-defined geometries and lend themselves to precise measurements of ultrasonic properties. This has allowed a well-controlled interlaboratory comparison of measurement procedures.

The level of agreement between the seven laboratories regarding ultrasonic speed is very good. The spread in values obtained is only about 0.3 per cent without correcting for small differences in temperature. The temperature dependency of ultrasonic speed in materials of the type used has been reported⁵ and is given by +1.6 m/sec°C. This may play a role, e.g., in explaining the slightly higher value measured at SRI International where the temperature used was about 3°C higher than that used at the other laboratories. An adjustment of the value obtained at SRI International, using the above temperature dependency, yields the value shown in parentheses in table 3 corresponding to 22.5°C.

Overall, the measurements of ultrasonic attenuation coefficients by the various techniques at the seven laboratories agree rather well with one another. To quantify the level of agreement in a reasonably unbiased way, a curve-fitting was performed with the results in figure 2. The curve-fitted relation used was $\alpha = \alpha_0 f^n$, where α_0 and n are constants, α is the attenuation coefficient, and f is the frequency. This is the most common relation found in the literature for fitting ultrasonic attenuation coefficients as a function of frequency. The

result of this curve-fitting process is referred to below as the "group curve-fitting." The measurements shown as four lines in figure 2 were each represented with three or four data points, resulting in a total of thirteen data points. For each of the three shorter lines, one point from each end and one from its center were used in the group curve-fitting process; the longest line (CDRH) was represented by four points equally spaced on the line with one point at each end. The results from the University of Wisconsin were represented by five points at five frequencies; the latter points correspond to the centers of the five rectangles plotted in figure 1. This group curve-fitting process yielded the relation $\alpha = 0.493 f^{0.961} \text{ dB cm}^{-1} \text{ MHz}^{-0.961}$, where f is the frequency.

The level of agreement of any value plotted in figure 2 with the group curve-fitting process is shown in table 4, where the percentage deviation from the group values is shown for each frequency involved. In the case of the four curve-fitted sets of data from individual laboratories, the three or four points selected for determining the group curve-fitting were used; the frequencies and ultrasonic attenuation coefficient values corresponding to the curve-fitted data are shown in table 4 in parentheses.

In many values given in table 4 the number of significant figures is too large for the likely accuracy. No uncertainty estimates are given in this study because of the complications which could result in trying to produce uniformity of error analysis from such a large number of participants and techniques. The reader is referred to cited publications for error estimates.

CONCLUSIONS

Nine different combinations of laboratories and measurement techniques were represented in this comparison. The level of interlaboratory agreement is very high for measurement of ultrasonic speed; therefore, one conclusion of this work is

Table 4. Level of Agreement of Ultrasonic Attenuation Coefficient Values Measured at Individual Laboratories with the Group Averages

Laboratory (and Technique if More than One)	Frequency (MHz)	L = Lab Attenuation Coefficient (dB/cm)	G = Group Curve-fitting Attenuation Coefficient (dB/cm)	Difference = (L - G)/G × 100 (%)
Univ. of Wisconsin	1.20	0.53	0.587	-9.8
	2.50	1.17	1.189	-1.6
	4.20	1.99	1.958	+1.6
	5.70	2.78	2.626	+5.9
	8.00	3.93	3.64	+8.1
Yale Univ.	1	0.417	0.493	-15.4
	3	1.341	1.417	-5.4
	5	2.20	2.32	-5.0
SRI International	(0.56)	(0.271)	0.282	-4.0
	(1.41)	(0.680)	0.684	-0.5
	(2.25)	(1.089)	1.075	+1.3
Univ. of Illinois (Radiation Force)	1.0	0.582	0.493	+18.1
	1.385	0.710	0.674	+5.3
	3.5	1.57	1.64	-4.5
Univ. of Illinois (Spectral Difference)	4.21	1.763	1.962	-10.2
	(3.3)	(1.60)	1.55	+3.0
	(4.8)	(2.41)	2.23	+8.3
Univ. of Michigan	(6.3)	(3.24)	2.89	+12.1
	(2.7)	(1.202)	1.281	-6.1
	(3.45)	(1.535)	1.621	-5.3
Univ. of Rochester	(4.2)	(1.87)	1.96	-4.5
	2.1	1.101	1.006	+9.5
	2.3	1.080	1.098	-1.6
CDRH (Radiation Force)	2.5	1.023	1.189	-14.0
	3.2	1.296	1.508	-14.0
	0.943	0.62	0.466	+33.1
	2.47	1.38	1.176	+17.4
CDRH (Spectral Difference)	2.96	1.77	1.399	+26.5
	3.51	1.81	1.648	+9.8
	(1)	(0.420)	0.493	+14.8
	(3)	(1.274)	1.417	-10.1
	(5)	(2.134)	2.315	-7.8
	(7)	(3.00)	3.20	-6.2

Note: The values shown in parentheses were obtained from the curve-fitting relations supplied by those laboratories. Percentage differences between values obtained from individual laboratories and the group average values are shown in the right-hand column.

that significant variations in this parameter found in the literature for a specific tissue parenchyma are probably not due to the measurement technique. It is not implied, however, that the uncertainty in ultrasonic speed measurements for tissues will be as small as in the case of the very regularly shaped test objects in this study. For example, suppose that the uncertainty in thickness of an approximately 5-cm-thick tissue sample is ± 0.5 cm, that the actual ultrasonic speed in the tissue sample is 1560 m/sec, and that the ultrasonic speed in the water is 1480 m/sec. Then, the method of measurement yielding Eq. [2] results in an uncertainty of ± 8.4 m/sec, or about 0.5 per cent.

The agreement between values of the ultrasonic

attenuation coefficients obtained for the various laboratories and techniques is reasonably good. Almost all of the measured values differ from the group average by considerably less than ± 20 per cent. However, because of the exponential nature of attenuation, an accuracy corresponding to an uncertainty of, say, ± 5 per cent may be required for some applications. One such application is the accurate determination of backscatter coefficients in tissues.

Table 4 should be particularly valuable to the participants in allowing them to assess their specific measurement techniques relative to a group norm. Other investigators who make attenuation measurements on tissues but were not part of this

study should similarly profit from these results where methods of measurement are similar to their own.

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