NONLINEAR PROPAGATION OF ULTRASOUND IN LIQUID MEDIA

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It is well known that acoustic phenomena are fundamentally non-linear, though a large class of acoustical phenomena can be dealt with by linearizing the equations of motion leading to acceptable and usable solutions(1,2). This is, of course, very convenient as the nonlinear relations are extremely difficult to deal with. However, it is necessary on occasion to have more details of the propagation processes, e.g., for a more profound understanding of the phenomena, than can be obtained from the linear theories. Thus, a program has been initiated to examine this question and preliminary measurements give an indication of the degree of nonlinearity of biological media.

The equation of motion or the wave equation for acoustic phenomena is obtained by invoking three constitutive equations, viz. an equation of continuity, a dynamical equation, and an equation of state of the medium in which the propagation is to take place. In Lagrangian coordinates, these are respectively:

\[ \rho = \rho_0 \left(1 + \frac{\partial \xi}{\partial x}\right)^{-1}, \quad p = \frac{\partial p}{\partial x}, \quad \text{and} \quad c^2 = \frac{\partial p}{\partial \rho} \tag{1} \]

Here, \( \rho \) and \( \rho_0 \) are the disturbed and undisturbed density, respectively, \( \xi \) is the particle displacement of the medium, \( p \) is the sound pressure, and \( x \) is the Lagrangian coordinate. Combining these equations leads to:

\[ \xi = c^2 / \left(1 + \frac{\partial \xi}{\partial x}^2 \xi_{uu}\right), \]
equation of motion obtained without approximation. It is seen if a Hooke's law relationship for the equation of state is
used, such that \( \frac{\partial p}{\partial \rho} \) is constant, and if the displacement
amplitude of the wave is sufficiently small such that \( \frac{\partial^2 \xi}{\partial x^2} \ll 1 \),
the very lossless wave equation is obtained, viz. \( \xi = c_0^2 \xi'' \).

Now consider the situation wherein the displacement amplitude
is negligible and the equation of state is more complex than the
Hooke's law relationship. The second point can be accommo-
dated by considering the equation of state to be expanded in a
series for the isentropic case such that:

\[
-p = \left( \frac{3 \rho}{\rho} \right) s, \rho = \rho_0
\]

\[
+ \frac{1}{2} \left( \frac{3 \rho}{\rho} \right)^2 s, \rho = \rho_0
\]

\[
+ \frac{1}{3} \left( \frac{3 \rho}{\rho} \right)^3 s, \rho = \rho_0
\]

\[...\]

Convenient to rewrite this in series form

\[
-p = p_o + \frac{B}{2A} s^2 + \frac{C}{3A} s^3 + ... ,
\]

where

\[
A = \rho_0 \left( \frac{\partial p}{\partial \rho} \right)_o s, \rho = \rho_0
\]

\[
B = 2 \left( \frac{\partial^2 p}{\partial \rho^2} \right)_o s, \rho = \rho_0
\]

\[
C = 3 \left( \frac{\partial^3 p}{\partial \rho^3} \right)_o s, \rho = \rho_0
\]

\[s = (\rho - \rho_o)/\rho_o \]

The speed of sound becomes:

\[
c^2 = c_0^2 \left[ 1 + \frac{B}{A} s + \frac{B}{2A} s^2 + ... \right]
\]

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and the equation of motion is, for the case where only the first
two terms in the series are retained,

\[
\ddot{\xi} = \frac{\frac{c_0^2}{(1 + \xi')^2} + \frac{B}{A}}{\xi''} .
\]

Here it is seen that for the situation where the displacement ampltude
cannot be neglected, the parameter \( B/A \) becomes a measure of
the nonlinearity of the propagating medium.

A consequence of the propagation described by this relation in
a fluid medium is that an originally sinusoidal, monochromatic
wave becomes distorted as it propagates, harmonics are generated, and
the amplitude of these harmonics is a function of the distance from the
source \( (3) \). The harmonics have zero amplitude at the source, increase
to a maximum value at a position from the source at which the effect
of absorption processes balances such harmonic production and propaga-
tion occurs as under linear conditions well beyond this point.

The quantity \( B/A \) can be approximated as:

\[
\frac{B}{A} = \frac{2p_2 c_0^3}{\pi f} \left[ \frac{p_2 (x)}{x p_1 (x)} \right] \left[ x p_1 (o) \right] - 2 ,
\]

where \( p_1 \) is the pressure amplitude of the fundamental, \( p_2 \) is the
pressure amplitude of the second harmonic, and \( x \) is propagation
distance \( (1, 4, 6) \). A method which determines these quantities has
the potential for yielding \( B/A \) for optically opaque media and for
in vivo preparations.

Values of \( B/A \) have been obtained by measuring the harmonic
content of pulses of sound at various distances from the source
(3 MHz fundamental) and for varying concentrations in water of
several biological macromolecules of interest \( (5) \). It has been found
that \( B/A \) appears to increase nearly linearly with increasing concen-
tration of proteins and to exhibit little dependence upon molecular
weight (in the range \( 10^2 \) to \( 10^6 \) Daltons). These data suggest that
the nonlinearity parameter increases with decreasing interparticle
distance. Thus it is clear that biological media exhibit
significant nonlinear ultrasonic propagation features which must be
studied in detail for deeper understanding leading to more sophisti-
cated diagnostic and therapeutic procedures.
REFERENCES


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PROPATHGATION OF ULTRASOUND IN SOLUTIONS OF BIOLOGICAL SUBSTANCES

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The values of ultrasound velocity and absorption coefficients in solutions are defined by different molecular interactions. There is a large amount of literature on the investigation of ultrasonic characteristics of solutions of biological substances by ultrasonic measurements. The dominant part of this paper is devoted to the measurements of the frequency dependences of ultrasound absorption and the investigation of fast relaxation processes. Substantially fewer works are related to the study of solutions by ultrasound velocity measurements. But such a proportion is a result of the absence of adequate velocity measurement methods and not due to the fact that the absorption coefficient is more informative than the velocity of ultrasound about the characteristics of a solution.

The purpose of this short review is to show the relations between acoustic characteristics of biological solutions and their molecular properties, with emphasis on the ultrasound velocity. There are two reasons for such intentions:

(a) The possibilities of ultrasound velocity measurements in an investigation of biological substances in solutions are much less known, and

(b) The value of ultrasound velocity reflects to a greater extent the molecular characteristics of solutions of biological substances than absorption.