NONLINEAR PROPAGATION OF ULTRASOUND IN LIQUID MEDIA

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It is well known that acoustic phenomena are fundamentally nonlinear, though a large class of acoustical phenomena can be dealt with by linearizing the equations of motion leading to acceptable and usable solutions(1,2). This is, of course, very convenient as the nonlinear relations are extremely difficult to deal with. However, it is necessary on occasion to have more details of the propagation processes, e.g., for a more profound understanding of the phenomena, than can be obtained from the linear theories. Thus, a program has been initiated to examine this question and preliminary measurements give an indication of the degree of nonlinearity of biological media.

The equation of motion or the wave equation for acoustic phenomena is obtained by invoking three constitutive equations, viz. an equation of continuity, a dynamical equation, and an equation of state of the medium in which the propagation is to take place. In Lagrangian coordinates, these are respectively:

$$\rho = \rho_o \left(1 + \frac{\partial \xi}{\partial x}\right)^{-1}, \quad p\ddot{\xi} = - \frac{\partial p}{\partial x}, \quad \text{and } c^2 = \frac{\partial p}{\partial \rho} \qquad (1)$$

Here, ρ and ρ_o are the disturbed and undistrubed density, respectively, ξ is the particle displacement of the medium, p is the sound pressure, and x is the Lagrangian coordinate. Combining these equations leads to:

$$\ddot{\xi} = c^2/\left(1 + \frac{\partial \xi}{\partial x}\right)^2 \xi'' ,$$

[e]quation of motion obtained without approximation. It is seen [...]if a Hooke's law relationship for the equation of state is [...]ved, such that $\partial p/\partial \rho$ is constant, and if the displacement [...]tude of the wave is sufficiently small such that $\partial \xi/\partial x \ll 1$, the [...]ary lossless wave equation is obtained, viz. $\ddot{\xi} = c^2\xi''$.

[N]ow consider the situation wherein the displacement amplitude [...]t negligible and the quation of state is more complex than the [...]e Hooke's law relationship. The second point can be accommo[...] by considering the equation of state to be expanded in a [...]'s series for the isentropic case such that:

$$p-p_o = \left(\frac{\partial p}{\partial \rho}\right)_{S,\rho=\rho_o} (\rho-\rho_o) + \frac{1}{2!}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{S,\rho=\rho_o} (\rho-\rho_o)^2 + \frac{1}{3!}\left(\frac{\partial^3 p}{\partial \rho^3}\right)_{S,\rho=\rho_o} (\rho-\rho_o)^3 + \ldots$$

[...] convenient to rewrite this in series form

$$p-p_o = As + \frac{B}{2!}s^2 + \frac{C}{3!}s^3 + \ldots ,$$

$$A = \rho_o \left(\frac{\partial p}{\partial \rho}\right)_{S,\rho=\rho_o} = \rho_o c_o^2 ;$$

$$B = \rho_o^2 \left(\frac{\partial^2 p}{\partial \rho^2}\right)_{S,\rho=\rho_o} ;$$

$$C = \rho_o^3 \left(\frac{\partial^3 p}{\partial \rho^3}\right)_{S,\rho=\rho_o}$$

$$s = (\rho-\rho_o)/\rho_o .$$

[t]he speed of sound becomes:

$$c^2 = c_o^2 \left[1 + \left(\frac{B}{A}\right) s + \left(\frac{B}{2A}\right) s^2 + \ldots\right]$$

and the equation of motion is, for the case where only the first two terms in the series are retained,

$$\ddot{\xi} = \frac{c_o^2}{(1+\xi')^{2+B/A}} \xi'' .$$

Here it is seen that for the situation where the displacement amplitude cannot be neglected, the parameter B/A becomes a measure of the nonlinearity of the propagating medium.

A consequence of the propagation described by this relation in a fluid medium is that an originally sinusoidal, monochromatic wave becomes distorted as it propagates, harmonics are generated, and the amplitude of these harmonics is a function of the distance from the source(3). The harmonics have zero amplitude at the source, increase to a maximum value at a position from the source at which the effect of absorption processes balances such harmonic production and propagation occurs as under linear conditions well beyond this point.

The quantity B/A can be approximated as:

$$\frac{B}{A} = \frac{2\rho_o c_o^3}{\pi f} \left[\frac{p_2(x)}{x p_1^2(o)}\right]\Bigg|_{x p_1(o) \to o} - 2 ,$$

where p_1 is the pressure amplitude of the fundamental, p_2 is the pressure amplitude of the second harmonic, and x is progagation distance(1,4,6). A method which determines these quantities has the potential for yielding B/A for optically opaque media and for _in vivo_ preparations.

Values of B/A have been obtained by measuring the harmonic content of pulses of sound at various distances from the source (3 MHz fundamental) and for varying concentrations in water of several biological macromolecules of interest(5). It has been found that B/A appears to increase nearly linearly with increasing concentration of proteins and to exhibit little dependence upon molecular weight (in the range 10^2 to 10^6 Daltons). These data suggest that the nonlinearity parameter increases with decreasing intersolute particle spacing. Thus it is clear that biological media exhibit significant nonlinear ultrasonic propagation features which must be studied in detail for deeper understanding leading to more sophisticated diagnostic and therapeutic procedures.

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PROPAGATION OF ULTRASOUND IN SOLUTIONS OF BIOLOGICAL SUBSTANC

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The values of ultrasound velocity and absorption coeffici in solutions are defined by different molecular interactions. There is a large amount of literature on the investigation of lar characteristics of solutions of biological substances by u sonic measurements. The dominant part of this paper is devote the measurements of the frequency dependences of ultrasound ab and the investigation of fast relaxation processes. Substanti fewer works are related to the study of solutions by ultrasoun velocity measurements. But such a proportion is a result of t absence of adequate velocity measurement methods and not due t fact that the absorption coefficient is more informative than velocity of ultrasound about the characteristics of a solution

The purpose of this short review is to show the relations between acoustic characterstics of biological solutions and th molecular properties, with emphasis on the ultrasound velocity There are two reasons for such intentions:

(a) The possibilities of ultrasound velocity measurements in investigation of biological substances in solutions are m less known, and

(b) The value of ultrasound velocity reflects to a greater ext the molecular characteristics of solutions of biological substances than absorption.