

## 0. INTRODUCTION: PHYSICAL DESCRIPTION OF ULTRASONIC FIELDS†

By Peter D. Edmonds and F. Dunn

### List of Symbols

$a$	subscript denoting amplitude
$A$	magnitude of $\alpha/f^2$ contributed by a relaxation process (except viscosity) at $f \ll f_r$
$B$	magnitude of $\alpha/f^2$ contributed by the viscosity relaxation process at $f \ll f_v$
$C_p$	specific heat of medium at constant pressure
$C'_p$	specific heat per unit mass of medium at constant pressure
$C_v$	specific heat of medium at constant volume
$D$	logarithmic decrement
$E_0$	acoustic energy density = energy stored per unit volume
$\Delta E_0$	energy loss per cycle due to absorption
$f$	frequency
$f_v$	relaxation frequency for viscosity
$g_1(\tau)$	distribution function for relaxation times
$G$	shear modulus of elasticity
$G^*(j\omega)$	complex shear modulus of elasticity
$G'(\omega), G''(\omega)$	real and imaginary parts of $G^*$
$G^\infty$	infinite frequency asymptote of $G'$
$I$	intensity
$I_{1+}, I_{1-}, I_{2+}$	intensities of incident, reflected, and transmitted waves
$j$	$\sqrt{-1}$
$k$	wave vector
$\mathcal{K}$	magnitude of $k$
$K$	bulk modulus of elasticity
$n$	integer
$p$	acoustic pressure
$P$	amplitude of $p$
$P_0$	ambient pressure

† Portions of this introduction have been adapted with permission from W. J. Fry and F. Dunn, *Ultrasound: Analysis and experimental methods in biological research*, in "Physical Techniques in Biological Research," (W. L. Nastuk, ed.), Vol. 4, pp. 265-275. Academic Press, New York, 1962; and from F. Dunn, P. D. Edmonds, and W. J. Fry, *Absorption and dispersion of ultrasound in biological media*, in "Biological Engineering" (H. P. Schwan, ed.), pp. 207-233. McGraw-Hill, New York, 1969.

$P_+, P_-$	} see Table IV
$P_{1+}, P_{1-}, P_{2+}, P_{2-}$	
$P_{\max}, P_{\min}$	
$P_{1i}, P_{1r}, P'_{1i}, P'_{1r}$	
$P_{2i}, P'_{2i}$	
$q$	general field variable
$Q$	amplitude of $q$
$Q_m$	quality factor = $\pi/D$
$e$	general spatial coordinate
$r_{2/1}, r_{3/1}, r_{2/3}$	} see Table IV
$r'_{1/1}, r'_{2/1}$	
$\mathcal{R}_a$	amplitude reflection coefficient
$\mathcal{R}_i$	intensity reflection coefficient
$s$	condensation = $(\rho - \rho_0)/\rho_0$
$S$	amplitude of condensation; entropy (as subscript)
SWR	standing wave ratio
$t$	time
$t_D$	decay time of field amplitude parameter in an absorbing medium
$T$	absolute temperature
$T$	shear stress
$T_i$	initial value of shear stress
$T^0$	asymptotic final value of shear stress
$\hat{T}$	amplitude of sinusoidal shear stress
$\hat{T}^0$	amplitude of asymptotic final value of shear stress
$\mathcal{T}_a$	amplitude transmission coefficient
$\mathcal{T}_i$	intensity transmission coefficient
$v$	wave propagation velocity; sound speed
$v^0$	limiting sound speed at zero frequency (compressional wave)
$v_t$	compressional wave speed
$v_s^0$	limiting shear wave speed at zero frequency
$v_1, v_2, v_3$	} see Table IV
$v_{11}, v_{12}, v_{s1}, v_{s2}$	
$\Delta v$	velocity dispersion
$x$	spatial coordinate
$x_3$	see Table IV
$Z_0, Z_1, Z_2$	characteristic acoustic impedance
$\alpha$	amplitude absorption coefficient
$\alpha_r$	generalized relaxational contribution to $\alpha$
$\alpha_v$	contribution to $\alpha$ from viscosity
$\beta_S$	adiabatic compressibility
$\beta_T$	isothermal compressibility
$\gamma$	ratio of specific heats = $C_p/C_v$
$\delta$	phase lag between acoustic pressure and particle velocity in an absorbing medium
$\eta(\omega)$	shear viscosity coefficient
$\eta^0$	limiting shear viscosity as frequency tends to zero
$\eta^*(j\omega)$	complex shear viscosity
$\eta'(\omega)$	real part of $\eta^*$
$\theta$	isobaric thermal expansion coefficient
$\theta_1, \theta_2, \theta'_1, \theta'_2$	see Table IV

$\Theta$	amplitude of temperature perturbation
$\lambda$	wavelength
$\lambda_3$	see Table IV
$\xi$	"particle" (elemental volume) displacement
$\xi_x$	particle displacement in $x$ direction
$\dot{\xi}_x$	particle velocity
$\ddot{\xi}_x$	particle velocity in $x$ direction
$\ddot{\xi}_x$	particle acceleration in $x$ direction
$\xi_{\pm}$	amplitudes of particle displacement for waves in the positive and negative directions
$\dot{\xi}_{\pm}$	amplitude of particle velocity
$\ddot{\xi}_{\pm}$	amplitude of particle acceleration
$\rho$	density
$\rho_0$	mean density
$\rho_1, \rho_2, \rho_3$	see Table IV
$\tau$	relaxation time
$\tau_a, \tau_b$	limits of relaxation time distribution
$\tau_v$	relaxation time for viscosity
$Y$	instantaneous temperature increment in medium
$\phi$	scalar displacement potential (irrotational)
$\Phi$	vector displacement potential (rotational)
$\psi$	scalar velocity potential
$\omega$	angular frequency
(dot over symbol)	differentiation with respect to time

## 0.1. Development of Propagation Relations

The propagation of an acoustic disturbance or the presence of an acoustic field in an elastic medium is characterized by changes in a number of the physical variables that describe the state of the system or medium. Examples of these variables are pressure, temperature, and density.

For a traveling, sinusoidal, plane wave propagating in the positive direction of the  $x$  axis (when no attenuation of the waves occurs because we assume absorption of energy by the medium is absent), the changes in the physical variables can each be expressed in the form of Eq. (0.1.1), provided that the medium responds linearly to the stresses imposed upon it.

$$q = Q \cos \omega(t - x/v) \quad \text{or} \quad q = \text{Re}\{Q \exp[j\omega(t - x/v)]\}. \quad (0.1.1)$$

In this equation  $q$  designates any one of the variables that undergoes sinusoidal change owing to the presence of the disturbance in the medium and  $Q$  designates the amplitude of the cyclic change in that variable;  $t$  and  $x$  are the time and space coordinates, respectively,  $\omega$  is the angular frequency ( $\omega = 2\pi f$ ),  $f$  the frequency, and  $v$  the free-field sound speed, i.e.,

the propagation speed of a plane wave traveling through a liquid medium of infinite extent. Equation (0.1.1) is one solution, namely, that representing a wave traveling in the positive  $x$  direction, of the one-dimensional elastic wave equation as it applies to an ideal, linear, homogenous, perfectly elastic (dissipationless), fluid medium

$$\partial^2 q / \partial t^2 = (1/v^2) \partial^2 q / \partial x^2. \quad (0.1.2)$$

In this equation  $q$  could represent the instantaneous displacement  $\xi$  of an element of volume of the medium. This approximation to the more general hydrodynamical equation is valid under conditions that permit linearization, that is, when the velocity amplitude  $\dot{\xi} = (\partial \xi / \partial t)_{\max}$  of the elementary volume is small in comparison with the speed of sound  $v$  and when the adiabatic compressibility  $\beta_s$ , which is the reciprocal of the adiabatic elastic bulk modulus  $K$ , is not significantly dependent on pressure over the range of pressure variations present in the acoustic field.

Since sound propagation is very close to an adiabatic process at most frequencies of interest, the adiabatic compressibility is a significant parameter in the description of sound propagation. It is related to the free-field sound speed for compressional waves as follows:

$$v^2 = v_t^2 = \frac{1}{\rho_0 \beta_s} = \frac{\gamma}{\rho_0 \beta_T} = \frac{C_p / C_v}{\rho_0 \beta_T}, \quad (0.1.3)$$

where  $\beta_s$  is the adiabatic compressibility of the medium and  $\rho_0$  the mean density of the medium. The sound speed can be expressed, as indicated in Eq. (0.1.3), in terms of the isothermal compressibility  $\beta_T$  by introducing the ratio of specific heats  $\gamma = C_p / C_v$ , where  $C_p$  and  $C_v$  are the specific heats of the medium at constant pressure and constant volume, respectively. Clearly, a measurement of the speed of a plane compressional wave can be interpreted immediately to yield the adiabatic compressibility of the medium if the density is known; and if the value of  $\gamma$  is also known, the isothermal compressibility can be determined.

Equation (0.1.2) is a special case of the more general wave equation that is applicable to three-dimensional propagation:

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{1}{\rho_0 \beta_s} \nabla^2 \xi. \quad (0.1.4)$$

Solutions of Eq. (0.1.4) include not only waves propagating in the positive  $r$  direction away from the origin but also those propagating in the negative  $r$  direction toward the origin. All are represented when the  $\pm$  sign is placed in the exponent for one-dimensional propagation, e.g.,

$$\xi = \Xi_{\pm}(r) \exp[j(\omega t \pm \mathbf{k} \cdot \mathbf{r})]. \quad (0.1.5)$$

The wave vector  $\mathbf{k}$  that appears in the solution is related to the angular frequency and the sound speed as

$$\mathbf{k} = \mathcal{K}\mathbf{n}; \quad \mathcal{K} = -\omega/v = 2\pi/\lambda; \quad v = f\lambda. \quad (0.1.6)$$

Equation (0.1.4) is itself a specialization, applicable to fluids of the type indicated, of the following wave equation describing propagation of disturbances in a dissipationless, isotropic, elastic solid:

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{K + 4G/3}{\rho_0} \nabla \nabla \cdot \xi - \frac{G}{\rho_0} \nabla \times \nabla \times \xi, \quad (0.1.7)$$

where  $K$  and  $G$  are, respectively, the bulk and shear moduli of elasticity of the medium.

It is possible to express the displacement vector as the sum of terms involving a scalar potential  $\phi$  and a vector potential  $\Phi$  as

$$\xi = \nabla \phi + \nabla \times \Phi \quad (0.1.8)$$

For irrotational motion, such as in a spherical wave, the vector potential  $\Phi = 0$  and only the scalar displacement potential  $\phi$  remains; that is,

$$\xi = \nabla \phi. \quad (0.1.9a)$$

The time derivative of the displacement potential is the velocity potential  $\psi$ , i.e.,

$$\partial \phi / \partial t = \psi; \quad \dot{\xi} = \nabla \psi. \quad (0.1.9b)$$

These potentials are fundamental functions (analogous to electric field potentials) in terms of which acoustic field parameters may be expressed. The specialization of Eq. (0.1.7) for fluids is obtained when the modulus of shear rigidity  $G$  is set equal to zero, which is true for lossless fluids, since the latter are characterized by an inability to support an elastic shear strain, and  $\Phi = 0$ .

Returning to a consideration of the simple plane wave propagating in an ideal isotropic elastic medium in the positive  $x$  direction, we can express the sinusoidally varying acoustic parameters in terms of the displacement potential or velocity potential and in terms of one another.

$$p = -\rho_0 \partial \psi / \partial t, \quad \dot{\xi}_x = (\nabla \psi)_x, \quad (0.1.10)$$

$$s = (\rho - \rho_0) / \rho_0 = \beta_s p, \quad (0.1.11)$$

$$Y = (T\theta / \rho_0 C'_p) p = (\gamma - 1)(\beta_s / \theta) p, \quad (0.1.12)$$

where  $s$  is the condensation or the fractional change in density,  $\rho$  the instantaneous density,  $Y$  the instantaneous temperature increment resulting from adiabatic compression of the medium,  $T$  the absolute temper-

TABLE I.

Parameter	Parameter symbol $q$	Amplitude symbol $Q$	$P$	$S$
Pressure	$p$	$P$	—	$\pm \rho_0 v^2$
Condensation	$s$	$S$	$\pm \frac{1}{\rho_0 v^2}$	—
Particle displacement	$\xi$	$\Xi$	$\pm \frac{1}{j\omega\rho_0 v}$	$\pm \frac{v}{j\omega}$
Particle velocity	$\dot{\xi}$	$\dot{\Xi}$	$\pm \frac{1}{\rho_0 v}$	$\pm v$
Particle acceleration	$\ddot{\xi}$	$\ddot{\Xi}$	$\pm \frac{j\omega}{\rho_0 v}$	$\pm j\omega v$
Temperature	$Y$	$\Theta$	$\pm \frac{1}{\theta} \left( \beta_T - \frac{1}{\rho_0 v^2} \right)$	$\pm \frac{\rho_0 v^2}{\theta} \left( \beta_T - \frac{1}{\rho_0 v^2} \right)$

<sup>a</sup> Multiply expression in the table by the column heading to obtain the relations equal to the amplitude quantities tabulated in the amplitude symbol column. Note that  $j = \sqrt{-1}$ . The relations apply to plane waves traveling in either direction. The upper sign applies to waves traveling in the positive direction and the lower sign to the negative direction [see Eq. (0.1.5)]. The amplitude of a change in any one physical parameter is equal to the amplitude of the change in any other physical parameter multiplied by the absolute value of the appropriate quantity in the table. A self-consistent set of units is used throughout the table (e.g., mks or cgs).

ature of the medium,  $\theta$  the isobaric thermal expansion coefficient, and  $C_p$  the heat capacity at constant pressure per unit mass. The interrelation of the acoustic field parameters is shown in Table I.

The method of detection and description of the field, in any specific case, may depend on the measurement of one or several of these parameters. The quantity  $\rho_0 v$ , the product of density and sound speed, which appears in many relations in the table, is known as the characteristic acoustic impedance of the medium  $Z_0$ ; that is,

$$Z_0 = \rho_0 v. \quad (0.1.13)$$

For plane traveling waves,  $Z_0$  is numerically equal to the specific acoustic impedance, which is defined as the ratio of the pressure  $p$  to the particle velocity  $\dot{\xi}$  at any point in the field. For other field configurations, including plane standing waves, the specific acoustic impedance differs numerically from  $\rho_0 v$  and is, in general, a function of position. It should also be noted that the characteristic acoustic impedance is dependent on

Relations between Amplitudes of the Various Physical Parameters<sup>a</sup>

$\Xi$	$\dot{\Xi}$	$\ddot{\Xi}$	$\Theta$
$\pm j\omega\rho_0 v$	$\pm\rho_0 v$	$\pm\frac{\rho_0 v}{j\omega}$	$\pm\frac{\theta}{(\beta_T - (1/\rho_0 v^2))}$
$\pm\frac{j\omega}{v}$	$\pm\frac{1}{v}$	$\pm\frac{1}{j\omega v}$	$\pm\frac{\theta}{\rho_0 v^2[\beta_T - (1/\rho_0 v^2)]}$
—	$\pm\frac{1}{j\omega}$	$\pm\frac{1}{\omega^2}$	$\pm\frac{\theta}{j\omega\rho_0 v[\beta_T - (1/\rho_0 v^2)]}$
$\pm j\omega$	—	$\pm\frac{1}{j\omega}$	$\pm\frac{\theta}{\rho_0 v[\beta_T - (1/\rho_0 v^2)]}$
$\pm\omega^2$	$\pm j\omega$	—	$\pm\frac{j\omega\theta}{\rho_0 v[\beta_T - (1/\rho_0 v^2)]}$
$\pm\frac{j\omega\rho_0 v}{\theta} \left(\beta_T - \frac{1}{\rho_0 v^2}\right)$	$\pm\frac{\rho_0 v}{\theta} \left(\beta_T - \frac{1}{\rho_0 v^2}\right)$	$\pm\frac{\rho_0 v}{j\omega\theta} \left(\beta_T - \frac{1}{\rho_0 v^2}\right)$	—

the type of wave that is propagating, since the speed of shear waves is different from that of compressional waves.

The intensity  $I$  of the sound wave is defined as the time average of the rate of propagation of energy through unit area normal to the direction of propagation; for plane traveling waves,  $I$  is related to field-parameter amplitudes by

$$I = P^2/2Z_0 = P\dot{\Xi}/2 = Z_0\ddot{\Xi}^2/2. \quad (0.1.14)$$

The energy density  $E_0$  of the wave motion at a specific position in the field is the sum of the kinetic energy per unit volume of the moving volume element and the potential energy per unit volume of compression (or expansion) of the element. For plane traveling waves, it is equal to the ratio of the intensity to the sound speed, i.e.,

$$E_0 = \rho_0\ddot{\Xi}^2/2 = I/v. \quad (0.1.15)$$

Root mean square (rms) quantities are not employed in the majority of publications in acoustics, and consequently the symbols in Eqs. (0.1.14) and (0.1.15) are the amplitudes of the acoustic field parameters. If rms values had been used, the factors 2 would have been eliminated from the equations.

As stated previously, linearizing of the hydrodynamical equations depends on two assumptions which can now be expressed symbolically as

$$\ddot{\Xi}/v \ll 1; \quad [(\beta_S)_{P_0+P} - (\beta_S)_{P_0-P}]/(\beta_S)_{P_0} \ll 1, \quad (0.1.16)$$

TABLE II. Numerical Example of Physical Parameters for Water

Material	$f$	$T$	$P_0$	$I$	$P$	$S$	$H$	$\dot{H}$	$\ddot{H}$	$\Theta$	$\dot{\Theta}$
To obtain results in:	MHz	°C	atm	W/cm <sup>2</sup>	atm		cm	cm/sec	cm/sec <sup>2</sup>	°C	
Multiply figures in table by:	1	1	1	1	1	10 <sup>-5</sup>	10 <sup>-6</sup>	1	10 <sup>6</sup>	10 <sup>-4</sup>	10 <sup>-5</sup>
To obtain results in:			N/m <sup>2a</sup>	W/m <sup>2</sup>	N/m <sup>2a</sup>		m	m/s	m/s <sup>2</sup>		
Multiply figures in table by:			1.013 × 10 <sup>5</sup>	10 <sup>4</sup>	1.013 × 10 <sup>5</sup>		10 <sup>-8</sup>	10 <sup>-2</sup>	10 <sup>4</sup>		
Water											
degassed and distilled	1	30	1	0.01	0.171	0.762	0.183	1.15	7.22	3.82	0.762
				1	1.71	7.62	1.83	11.5	72.2	38.2	7.62
				100	17.1	76.2	18.3	115	722	382	76.2

<sup>a</sup> N/m<sup>2</sup> ≡ Pascal (Pa).



where  $P_0$  represents the ambient pressure in the absence of a sound wave. Nonlinear or second-order effects still may be of importance for values of  $\Xi/v$  smaller, for example, than 0.01, but the linearized equations constitute a good first approximation for calculating values of the physical parameters when this numerical limit is placed on the interpretation of the symbol  $\ll 1$ .

Table II shows values of the numerical magnitudes of the acoustic field parameters for a plane traveling wave, when the propagation medium is water, for representative intensity values of the wave spanning four orders of magnitude. It may be noted in particular that the temperature excursion in water is small and that this parameter is entirely unrelated to the monotonic rise in temperature of the specimen that occurs when energy is absorbed by the specimen. However, even for low-amplitude ultrasonic waves, which may be used as a probe to measure the response of a system to an extremely small perturbation, the pressure amplitude may be comparable to one atmosphere, and the amplitude of the particle acceleration can be exceedingly high and give rise to significant local stresses.

Table III lists values for the various characteristic constants of a number of materials of general utility. These data may be used in connection with the relations appearing in Table I to obtain numerical values of field parameters such as those listed in Table II. It is usually convenient to express the intensity in watts per square centimeter and the acoustic pressure amplitude in atmospheres. However, for calculations using the expressions of Table I, the intensity should be expressed in ergs per square centimeter per second and the pressure amplitude in dynes per square centimeter if the other parameters are expressed in the indicated units. Equivalent mks units may also be used.

## 0.2. Reflection and Refraction

Reflection and refraction of acoustic waves occur in a manner analogous to that for electromagnetic waves, and many of the concepts that arise in the theory of transmission lines are applicable in "one-dimensional" situations. The formulas listed in Table IV are for media within which no acoustic absorption occurs and for which the normals to the planar wave fronts and the normals to the interfaces lie in the same plane.

*Case 1.* Reflection and transmission occur at a single interface between two media. The reflection coefficient  $\mathcal{R}_a$ , the transmission coefficient  $\mathcal{T}_a$ , and the standing wave ratio (SWR) for waves incident on the interface

TABLE IIIA. Physical Constants of Various Materials

Material	T	P <sub>0</sub>	ρ <sub>0</sub>	v	ρ <sub>0</sub> <sup>v</sup>	C <sub>p</sub> /C <sub>v</sub>	β <sub>T</sub>	θ	α
To obtain results in:	°C	atm	g/cm <sup>3</sup>	cm/sec	g/(cm <sup>3</sup> sec)	1	cm <sup>2</sup> /d	(°C) <sup>-1</sup>	Np/cm
Multiply figures in table by:	1	1	1	10 <sup>5</sup>	10 <sup>5</sup>	1	10 <sup>-12</sup>	10 <sup>-6</sup>	1
To obtain results in:		N/m <sup>2a</sup>	kg/m <sup>3</sup>	m/s	kg/(m <sup>3</sup> s)		m <sup>2</sup> /N <sup>a</sup>		Np/m
Multiply figures in table by:		1.013 × 10 <sup>5</sup>	10 <sup>3</sup>	10 <sup>3</sup>	10 <sup>6</sup>		10 <sup>-11</sup>		10 <sup>2</sup>
Water									
Degassed, distilled	0	1	0.999841	1.4027	1.4025	1.000583	50.86	-5.89	
α proportional to f <sup>2b</sup>	10	1	0.999701	1.4476	1.4472	1.001085	47.79	+9.45	
	20	1	0.998207	1.4827	1.4800	1.00656	45.86	21.19	25 × 10 <sup>-5</sup>
	30	1	0.995651	1.5094	1.5028	1.01526	44.76	30.75	
	40	1	0.992220	1.5292	1.5173	1.02575	44.20	38.93	
	0	136	0.9941	1.4245	1.4161	1.00012	49.58	2.01	
	10	136	0.9946	1.4700	1.4621	1.00356	46.69	15.09	
	20	136	0.9961	1.5057	1.4998	1.01041	44.74	25.10	
	30	136	0.9986	1.5329	1.5308	1.01827	43.40	34.05	
	40	136	1.0019	1.5531	1.5560	1.02672	42.48	40.92	
Water Solutions									
0.9% normal saline <sup>c</sup>	0	1	1.00668	1.4134	1.4228			1.98	
α proportional to f <sup>2b</sup>	10	1	1.00631	1.4582	1.4674			8.46	
	20	1	1.00460	1.4932	1.5001			23.89	25 × 10 <sup>-5</sup>
	30	1	1.00189	1.5198	1.5268			29.94	
	40	1	0.99837	1.5394	1.5369			40.07	

Oils

Castor, at 30°C  
 $\alpha$  proportional to  $f^{3/2}$ <sup>a,d</sup>

0	1	0.972	1.580	1.536	0.26
10	1	0.960	1.536	1.474	0.16
20	1	0.952	1.494	1.422	0.096
30	1	0.946	1.452	1.374	0.057
40	1	0.941	1.411	1.328	0.037
0	1	1.124	1.446	1.625	
10	1	1.112	1.409	1.567	0.135
20	1	1.102	1.378	1.518	0.070
30	1	1.095	1.349	1.477	0.040
40	1	1.089	1.321	1.438	0.024
		2.70	6.42	17.3	
		2.5-3.4	4.6-6.8	12-18	

Phenylated silicone

Dow-Corning No. 710

$\alpha$  proportional to  $f^2$  to  $\sim 20$  MHz<sup>b,e</sup>

Aluminum (rolled)

Ceramics (approximate range)

Glasses

Borate crown (light)

Pyrex (702)

Silicate flint (heavy)

Silica (fused)

Stainless steel (347)

2.24	5.10	11.4
2.32	5.64	13.1
3.88	3.98	15.4
2.2	5.97	13.1
7.91	5.79	45.8

TABLE IIIB. Physical Constants of Biological Media<sup>a</sup>

Material	T	P <sub>0</sub>	ρ <sub>0</sub>	v	ρ <sub>0</sub> v	α
To obtain results in:	°C	atm	g/cm <sup>3</sup>	cm/sec	g/(cm <sup>2</sup> sec)	Np/cm
Multiply figures in table by:	1	1	1	10 <sup>5</sup>	10 <sup>5</sup>	1
To obtain results in:		N/m <sup>2</sup> <sup>a</sup>	kg/m <sup>3</sup>	m/sec	kg/(m <sup>2</sup> sec)	Np/m
Multiply figures in table by:		1.013 × 10 <sup>5</sup>	10 <sup>3</sup>	10 <sup>3</sup>	10 <sup>6</sup>	10 <sup>2</sup>
Central nervous system <sup>b, c</sup>						} See Table IIIC
Brain (average)	37	1	1.03	1.51	1.56	
Soft parenchymal tissues, e.g., liver, kidney (average)	37	1	1.05	1.56	1.64	
Muscle (skeletal) <sup>b, d, h</sup>	37	1	1.07	1.57	1.68	0.13
Fat <sup>b, h</sup>	37	1	0.97	1.44	1.40	0.05
Bone						
Skull (human) <sup>f</sup>	37	1	1.7	3.36	6.0	0.4
Frequency (MHz)						0.9
0.8		1				1.7
1.2		1				3.2
1.6		1				4.2
1.8		1				5.3
2.25		1				7.8
3.5		1				

TABLE III. Ultrasonic Absorption in Biological Tissues<sup>f</sup>

Tissue	Frequency $f$ (MHz)							Regression analysis fit	
	0.5	0.7	1	3	4	7	$\alpha$	R	
Brain	—	0.014 ± 0.003	0.029 ± 0.004	—	—	0.23 ± 0.09	0.024 $f^{1.18}$	0.993	
Heart	—	0.018 ± 0.009	0.033 ± 0.006	—	—	0.21 ± 0.03	0.028 $f^{1.04}$	0.995	
Kidney	—	0.017 ± 0.007	0.033 ± 0.004	—	—	0.20 ± 0.002	0.028 $f^{1.02}$	0.994	
Liver	0.010 ± 0.006	0.020 ± 0.003	0.023 ± 0.004	—	0.14 ± 0.03	0.24 ± 0.02	0.026 $f^{1.17}$	0.995	
Tendon	0.050 ± 0.03	0.16 ± 0.1	0.11 ± 0.04	0.53 ± 0.2	0.75 ± 0.4	1.4 ± 0.5	0.14 $f^{1.17}$	0.973	
Testis	0.0078 ± 0.002	0.0085 ± 0.001	0.015 ± 0.003	—	0.079 ± 0.02	0.12 ± 0.02	0.015 $f^{1.11}$	0.995	

<sup>a</sup> N/m<sup>2</sup> = Pascal (Pa); (m<sup>2</sup>/N) = (Pa<sup>-1</sup>).

<sup>b</sup> Values of  $\alpha$  for 1 MHz.

<sup>c</sup> Measurements of W.D. Wilson, U.S. Naval Ordnance Laboratory.

<sup>d</sup> Indicated power dependence holds over entire range of measurements from 400 kHz to 500 MHz at 30°C.

<sup>e</sup> Measurements at 26°C over the frequency range 1 to 2000 MHz indicate (assuming negligible velocity dispersion) the presence of a single relaxation process centered at 40 MHz.

<sup>f</sup> Extensive tabular and graphical data are given by Goss *et al.*,<sup>1</sup> Goss *et al.*,<sup>2</sup> Chivers and Parry,<sup>3</sup> and Bamber and Hill.<sup>4</sup>

<sup>g</sup>  $\alpha$  varies with direction of sound propagation relative to fiber orientation.

<sup>h</sup>  $\alpha$  proportional to frequency.

<sup>i</sup> Absorption coefficient listed for bone includes effects of reflections at interfaces within the bone structure. More extensive data are given by Fry and Barger.<sup>5</sup>

<sup>j</sup> Goss *et al.*<sup>2</sup> The absorption coefficient  $\alpha$  ± standard deviation is given in nepers per centimeter at 37°C.

TABLE IIID. Ultrasonic Attenuation in Biological Tissues<sup>a</sup>

Tissue	Frequency $f$ (MHz)						Regression analysis fit	
	0.5	0.7	1	3	4	7	$A$	$R$
Brain	0.032	0.047	0.07	0.24	0.34	0.64	$0.07f^{1.14}$	0.822
Heart	0.060	0.086	0.13	0.41	0.56	1.0	$0.13f^{1.07}$	0.98
Kidney	0.049	0.070	0.10	0.34	0.47	0.87	$0.10f^{1.09}$	0.973
Liver	0.038	0.055	0.08	0.29	0.40	0.75	$0.08f^{1.13}$	0.934
Tendon	0.33	0.42	0.56	1.3	1.6	2.5	$0.56f^{0.763}$	0.998

<sup>a</sup> Goss *et al.*<sup>2</sup> The attenuation coefficient  $A$  is given in nepers per centimeter at 37°C.

from medium 1 are functions only of the ratio of the characteristic acoustic impedances of the two media,  $r_{2/1} = \rho_2 v_2 / \rho_1 v_1$ .

For the partial reflection at normal incidence, the complete expression for the pressure variation for a sinusoidal disturbance of infinite extent can be represented by the summation of two waves, one traveling in the positive and the second in the negative  $x$  direction:

$$p = P_+ \exp[j\omega(t - x/v)] + P_- \exp[j\omega(t + x/v)], \quad (0.2.1)$$

where  $P_+$  is the amplitude of the pressure wave traveling in the positive direction and  $P_-$  the amplitude of a similar wave traveling in the negative direction. The standing wave ratio in either medium may be defined as

$$\text{SWR} = \frac{|P_{\max}|}{|P_{\min}|} = \frac{1 + |P_-/P_+|}{1 - |P_-/P_+|}, \quad (0.2.2)$$

where  $P_{\max}$  is the maximum value of the pressure amplitude in the field of interference of the incident and reflected waves and  $P_{\min}$  the minimum value of the pressure amplitude.

A distinction is required between reflection and transmission coefficients referring to the amplitude of the disturbance and those referring to the power carried by the acoustic waves. The coefficients are defined, respectively, as the ratios of the amplitudes or the intensities of the re-

<sup>1</sup> S. A. Goss, R. L. Johnston, and F. Dunn, *J. Acoust. Soc. Am.* **64**, 423-467 (1978).

<sup>2</sup> S. A. Goss, L. A. Frizzell, and F. Dunn, *Ultrasound Med. Biol.* **5**, 181-186 (1979).

<sup>3</sup> R. C. Chivers and R. J. Parry, *J. Acoust. Soc. Am.* **63**, 940-953 (1978).

<sup>4</sup> J. C. Bamber and C. R. Hill, *Ultrasound Med. Biol.* **5**, 149-157 (1979).

<sup>5</sup> F. J. Fry and J. E. Barger, *J. Acoust. Soc. Am.* **63**, 1576-1590 (1978).

TABLE IV. Pressure Amplitude of Reflected and Transmitted Waves for Various Combinations of Media<sup>a</sup>

Configuration	Definition	Formula
<p><i>Case 1.</i> Wave in medium 1 at normal incidence on boundary between medium 1 and medium 2. No energy returned to interface from medium 2. No absorption in media.</p>	$r_{211} = \frac{\rho_2 v_2}{\rho_1 v_1}$	$\mathcal{P}_a = \left  \frac{P_{1-}}{P_{1+}} \right  = \frac{1 - r_{211}}{1 + r_{211}}$ $\mathcal{T}_a = \left  \frac{P_{2+}}{P_{1+}} \right  = \frac{2r_{211}}{1 + r_{211}}$ $(\text{SWR})_1 = \begin{cases} r_{211} & \text{when } r_{211} > 1 \\ 1/r_{211} & \text{when } r_{211} < 1 \end{cases}$
<p><i>Case 2.</i> Wave in medium 1 at normal incidence. Slab of medium 3 interposed between media 1 and 2. No energy returned to interface from medium 2. No absorption in media.</p>	$r_{211} = \frac{\rho_2 v_2}{\rho_1 v_1}$ $r_{311} = \frac{\rho_3 v_3}{\rho_1 v_1}$ $r_{213} = \frac{\rho_2 v_2}{\rho_3 v_3}$	$\mathcal{P}_a = \left  \frac{P_{1-}}{P_{1+}} \right  = \left[ \frac{4r_{211}}{(r_{211} + 1)^2 \cos^2(\omega x_3 / v_3) + (r_{311} + r_{213})^2 \sin^2(\omega x_3 / v_3)} \right]^{1/2}$ $\mathcal{T}_a = \left  \frac{P_{3+}}{P_{1+}} \right  = \left[ \frac{(4r_{211})^2}{(r_{211} + 1)^2 \cos^2(\omega x_3 / v_3) + (r_{311} + r_{213})^2 \sin^2(\omega x_3 / v_3)} \right]^{1/2}$

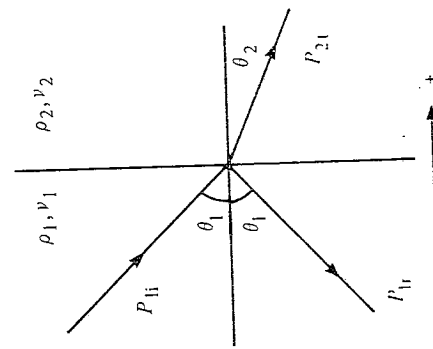
<sup>a</sup> Only the ratios of the magnitudes of the pressure amplitudes are shown in the table, i.e., phase factors are not shown.

TABLE IV (Continued)

Configuration	Definition	Formulas
---------------	------------	----------

Case 3. Wave in medium 1 incident at angle  $\theta_1$  to normal to boundary between media 1 and 2. Fluid media. No energy returned to interface from medium 2. No absorption in media.

$$r_{211} = \frac{\rho_2 v_2}{\rho_1 v_1}$$



$$R_a = \frac{|P_{1r}|}{|P_{1i}|} = \frac{(\cos \theta_2 / \cos \theta_1) - r_{211}}{(\cos \theta_2 / \cos \theta_1) + r_{211}}$$

$$T_a = \frac{|P_{2t}|}{|P_{1i}|} = \frac{2r_{211} \cos \theta_1}{\cos \theta_2 + r_{211}}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$$



Case 4. Longitudinal or shear wave in medium 1 incident at angle  $\theta_1$  to normal to boundary. Solid or viscoelastic media. Mode conversion with generation of shear and longitudinal waves in media 1 and 2. No energy returned to interface from medium 2. Absorption in media sufficiently small for impedances of media to be approximated by their real parts.<sup>b</sup>

$$r_{211} = \frac{\rho_2 v_{s2}}{\rho_1 v_{l1}}$$

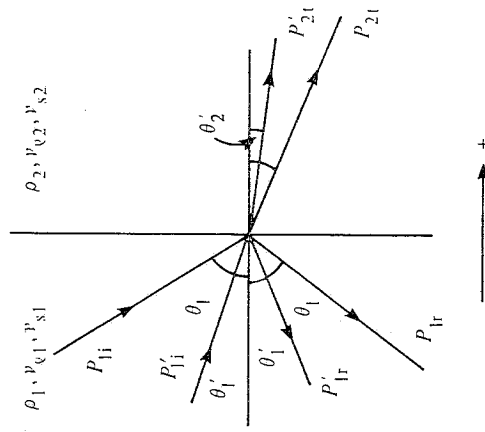
$$r'_{211} = \frac{\rho_2 v_{s2}}{\rho_1 v_{l1}}$$

$$r'_{111} = \frac{\rho_1 v_{s1}}{\rho_1 v_{l1}}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_{l1}}{v_{l2}}$$

$$\frac{\sin \theta_1}{\sin \theta'_2} = \frac{v_{l1}}{v_{s2}}$$

$$\frac{\sin \theta_1}{\sin \theta_1} = \frac{v_{l1}}{v_{s1}}$$



<sup>b</sup> See Muskat and Meres<sup>6</sup> for expressions for ratios of normal components of wave amplitudes.

<sup>6</sup> M. Muskat and M. W. Meres, *Geophysics* 5, 115 (1940).

flected and transmitted waves to the amplitude or intensity of the incident wave, i.e.,

$$\mathcal{R}_a = \frac{P_{1-}}{P_{1+}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \quad \mathcal{T}_a = \frac{P_{2+}}{P_{1+}} = \frac{2Z_2}{Z_2 + Z_1}, \quad (0.2.3)$$

$$\mathcal{R}_I = \frac{I_{1-}}{I_{1+}} = \left( \frac{P_{1-}}{P_{1+}} \right)^2 = \mathcal{R}_a^2, \quad \mathcal{T}_I = \frac{I_{2+}}{I_{1+}} = \frac{Z_1}{Z_2} \mathcal{T}_a^2. \quad (0.2.4)$$

In these expressions the subscript a designates amplitude coefficients and the subscript I power or intensity coefficients. Conservation of energy requires that the sum of the power reflection and transmission coefficients should always equal unity whereas the sum of the amplitude coefficients is not in general equal to unity.

$$\mathcal{R}_I + \mathcal{T}_I = 1. \quad (0.2.5)$$

The coefficients described here as transmission coefficients are frequently described as absorption coefficients in experiments involving irradiation of a specimen. This difference in viewpoint arises from the fact that in irradiation experiments interest is confined to measuring the standing wave ratio in the medium situated in front of the specimen. Any energy transmitted into the specimen is therefore effectively lost or appears to be absorbed. From the point of view of the properties of the interface between two media, this "lost" energy is merely transmitted into the second medium. The term "absorption coefficient" will be reserved for later use when a study is made of the attenuation of the transmitted wave amplitude mechanisms operative within the second medium.

*Case 2.* A slab of a third medium of thickness  $x_3$  is interposed between two media. The reflection coefficient  $\mathcal{R}_a$  and the transmission coefficient  $\mathcal{T}_a$  are functions of the ratios of the characteristic impedances and of the quantity  $\omega x_3/v_3$  (equal to  $2\pi x_3/\lambda_3$ ), which is determined by the ratio of the thickness of medium 3 and the wavelength in it. If the characteristic impedance of medium 3 is intermediate between those of media 1 and 2, then the transmission coefficient can be maximized by choosing thickness  $x_3$  to satisfy the relation

$$x_3/\lambda_3 = (2n - 1)/4, \quad n = 1, 2, 3, \dots \quad (0.2.6)$$

The transmission coefficient then becomes

$$\mathcal{T}_a = |2r_{2/1}/(r_{3/1} + r_{2/3})|; \quad (0.2.7)$$

that is, for maximum transmission, the best choice of thickness that can be made for any interposed material (if its characteristic acoustic impedance has any value between those of the other media) is one-quarter

wavelength or odd multiples thereof. In addition, if one is free to choose the interposed material so that its characteristic acoustic impedance is optimum for transmitting the acoustic energy, then the reflected wave in medium 1 can be eliminated by choosing the intermediate material so that

$$(\rho_3 v_3)^2 = (\rho_1 v_1)(\rho_2 v_2). \quad (0.2.8)$$

If the characteristic acoustic impedance of medium 3 does not have a value between those of the other two media, then the optimum choice of thickness for the slab to obtain the maximum value to the transmission coefficient is an integral multiple of a half-wavelength, i.e.,

$$x_3/\lambda_3 = n(\frac{1}{2}), \quad n = 1, 2, 3, \dots \quad (0.2.9)$$

The transmission coefficient then becomes identical with Eq. (0.2.3). If media 1 and 2 have nearly equal characteristic acoustic impedances that are less than that of medium 3 and if the thickness of the interposed slab satisfies the relation  $r_{3/1}(\omega x_3/v_3) \leq \frac{1}{10}$ , then the transmission coefficient does not differ from that of case 1 by more than 1%. If the characteristic acoustic impedance of medium 3 is less than that of media 1 and 2, then  $r_{2/3}$  should be used in place of  $r_{3/1}$  in the foregoing inequality.

*Case 3.* A plane wave is incident at any angle  $\theta_1$  on the plane interface between two fluid media. The angle of refraction  $\theta_2$  is a function of the angle of incidence and the ratio of the velocities of sound in the two media. The pressure transmission and reflection coefficients also involve the ratio of characteristic impedances. If  $\sin \theta_1 > v_1/v_2$ , then the incident wave is totally reflected, and there is no propagation of a refracted wave in medium 2. It should also be observed from the form of the reflection coefficient that there is no reflected wave if the ratio of velocities satisfies either of the relations  $\rho_2/\rho_1 > v_1/v_2 > 1$  or  $\rho_2/\rho_1 < v_1/v_2 < 1$  and if the angle of incidence satisfies the relation

$$\sin \theta_1 = \frac{r_{2/1}^2 - 1}{r_{2/1}^2 - (v_2/v_1)^2}. \quad (0.2.10)$$

*Case 4.* When the waves are incident obliquely and both media are solid or viscoelastic, the effects of shear rigidity are exhibited. The boundary conditions to be satisfied are: continuity of pressure and continuity of the normal and parallel components of particle displacement. The condition on the parallel component is satisfied by the occurrence of shear waves in one or both media. In the configuration of Table IV, the direction of polarization of the shear waves is in the plane of the diagram. An obliquely incident longitudinal wave generates reflected and refracted longitudinal waves and, in addition, reflected and refracted shear waves. An incident shear wave polarized in the plane of the diagram generates a

similar set of four waves. A shear wave obliquely incident as indicated and polarized perpendicularly to the diagram, that is, parallel to the interface, will generate only refracted and reflected shear waves of the same polarization since there is no component of motion perpendicular to the interface.<sup>7</sup> If either medium behaves as an ideal fluid, then it does not support shear wave propagation.<sup>8</sup> Muskat and Meres<sup>6</sup> derived expressions for the ratios of the components of displacements perpendicular to an interface between two perfectly elastic solids; their results may be used not only for perfectly elastic solids but also, with caution, for those viscoelastic solids exhibiting small absorption of energy. Absorption can be regarded as the result of independent processes influencing the wave amplitudes during propagation toward and away from the interface.

The formulas given in this section (Cases 1-4) are important, for example, in calculating, at least approximately, sound speed values from standing wave data, the magnitude of the effect of the reflected acoustic energy on driving transducers, the amplitude of the waves reflected at tissue interfaces, the accuracy of geometric placement or localization of a beam focus deep in tissue, etc. The formulas are also useful in the design of ultrasonic instruments where considerations of energy transfer from the transducer to the material of interest arise. More complicated configurations of materials and interfaces may arise in practice. The effects on the field of absorption within a medium will be considered in the following sections dealing with the physical mechanisms of absorption.

### 0.3. Absorption

When an ultrasonic wave propagates through any real medium, energy is absorbed from the wave and converted into heat. The rate of heat production in a selected volume of a medium in which such a field exists is determined by the amplitude, frequency, and spatial distribution of the field parameters. A variety of different mechanisms may play a role in the conversion of sonic energy into heat.

The occurrence of absorption modifies the phenomenological description of lossless plane wave propagation by the introduction of an absorption coefficient into Eq. (0.1.1), i.e.,

$$q = Q \exp(-\alpha x) \operatorname{Re}\{\exp[j\omega(t - x/v)]\}, \quad (0.3.1)$$

where  $\alpha$  is the amplitude absorption coefficient per unit distance. The in-

<sup>7</sup> M. R. Redwood, "Mechanical Waveguides." Pergamon, Oxford, 1960.

<sup>8</sup> W. G. Mayer, *IEEE Trans. Sonics Ultrason.* SU-11, 1 (1964).

tensity absorption coefficient per unit distance is equal to  $2\alpha$ . The fractional energy *loss* per unit volume per cycle is

$$\frac{\Delta E_0}{E_0} = \frac{1}{E_0} \int_0^{1/f} (P_0 + p) dV = 2\alpha\lambda, \quad (0.3.2)$$

where  $E_0$  is the energy stored per unit volume. This quantity may also be expressed in terms of a quality factor  $Q_m$  or the logarithmic decrement  $D$  of a field parameter per cycle, defined by Eq. (0.3.3), both of which are commonly used to describe the behavior of acoustic or electrical resonators:

$$\frac{\Delta E_0}{\pi E_0} = \frac{1}{Q_m} = \frac{D'}{\pi} = \tan \delta = \frac{1}{\pi f t_D} = \frac{\alpha\lambda}{\pi} \quad \text{for } \alpha \ll \frac{\omega}{v}, \quad (0.3.3)$$

where  $\delta$  is the angle of lag between a perturbation applied to the medium and an appropriate response parameter and  $t_D$  is the decay constant of a field-amplitude parameter. Absorption occurs in a homogeneous medium when the changes in density are not in time phase with the changes in pressure, i.e., when the time at which the maximum pressure occurs differs from the time at which the maximum density occurs. This type of behavior is produced by a variety of mechanisms classified under two general categories: relaxation and hysteresis for homogeneous media; relative motion and bubble mechanisms, for inhomogeneous media.

### 0.3.1. Relaxation Processes

It will be convenient to discuss relaxation phenomena first in terms of a specific example. The relaxation mechanism that is related to the shear viscosity of the medium is chosen for this purpose. If viscosity is the only mechanism responsible for absorption of a traveling, plane, compressional wave, then the absorption coefficient is given by

$$\alpha_v = \frac{2\pi^2 f^2}{\rho_0 v_l^3} \cdot \frac{4}{3} \eta \equiv B f^2 \quad \text{for } \frac{\alpha_v \lambda}{2\pi} \ll 1, \quad (0.3.4)$$

where  $\eta$  is the shear viscosity coefficient of the medium and  $B = 8\pi^2 \eta / 3\rho_0 v_l^3$  is the classical absorption parameter related to viscosity. In many nonmetallic liquids, particularly those which are associated and thus exhibit appreciable viscosity, it is found that the measured absorption coefficient is approximately described by the classical absorption expression (0.3.4) within a factor of about 3. In other cases the classical and measured absorption coefficient values differ by orders of magnitude. Consider first a hypothetical liquid for which Eq. (0.3.4) accurately describes the measured absorption coefficient at lower frequencies. At

higher frequencies it appears that the absorption coefficient should increase in proportion to the square of the frequency, assuming that the viscosity remains constant while the frequency is allowed to increase. This prediction is approximately true over only a limited frequency range for which the effective value of the viscosity coefficient is the same as the value at low frequencies, that is, under "static" conditions. As the frequency increases, the effective viscosity decreases monotonically toward zero, owing to the finite time required for the transfer of momentum between adjacent regions of the medium.

Under nonequilibrium conditions the instantaneous shear stress  $\mathbf{T}$  across a planar element at any position in a medium is not equal to the "static" value given by the product of the "static" shear viscosity coefficient and the space gradient of the particle velocity, but this product constitutes an asymptotic value  $\mathbf{T}^0$  toward which  $\mathbf{T}$  tends as time increases. The simplest assumption regarding the approach to the "static" value is that the rate is proportional to the difference between the instantaneous value and the "static" value; i.e.,

$$\frac{\partial \mathbf{T}}{\partial t} = \frac{1}{\tau_v} \left( \eta^0 \frac{\partial \xi}{\partial x} - \mathbf{T} \right), \quad (0.3.5)$$

where  $\tau_v$  is the proportionality constant which is in the nature of a time constant, the relaxation time, and  $\eta^0$  is the low-frequency viscosity coefficient.

Consider a step function change of velocity to be imposed on the  $x$  boundary of the system. Since transfer of momentum in the  $x$  direction is necessary to change the internal stress conditions, the shear stress  $\mathbf{T}(t)$  will not rise to  $\mathbf{T}^0$  immediately but will tend asymptotically to this limit. An approximate solution to Eq. (0.3.5) is obtained by equating the instantaneous viscous and inertial forces and by regarding  $\eta^0 \partial \xi / \partial x$  as the time-independent stress  $\mathbf{T}^0$ ; Eq. (0.3.5) becomes

$$\frac{\partial \mathbf{T}}{\partial t} = \frac{1}{\tau_v} (\mathbf{T}^0 - \mathbf{T}), \quad (0.3.6)$$

yielding the solution

$$\mathbf{T} = \mathbf{T}^0 - (\mathbf{T}^0 - \mathbf{T}_1) \exp(-t/\tau_v),$$

where  $\mathbf{T}_1$  was the initial value of  $\mathbf{T}$ . Consequently the shear stress in this hypothetical experiment increases approximately exponentially toward  $\mathbf{T}^0$  with time constant  $\tau_v$ . A time delay is exhibited in the response of the liquid, where "response" refers to the changes in the time-dependent stress  $\mathbf{T}(t)$ , after imposition of the step function.

The concept of  $\mathbf{T}^0$  as a stress toward which the instantaneous stress in

the liquid tends, even though it may never reach it, can be helpful in visualizing the response of a viscous liquid to a *sinusoidal* change in the imposed strain rate. In this case it is evident that a time delay in the response of the liquid will result in a phase delay between stress and imposed strain rate. Such a phase delay is characteristic of relaxing systems subjected to sinusoidal perturbation.

When the ultrasonic perturbation is a sinusoidal change in the rate of strain, the following substitutions are made in Eqs. (0.3.5) and (0.3.6):

$$\begin{aligned} \mathbf{T} &= \mathbf{T}(x)e^{j(\omega t + \delta)}, & \dot{\xi} &= \dot{\Xi}(x)e^{j\omega t}, \\ \mathbf{T}^0 &= \hat{\mathbf{T}}^0(x)e^{j\omega t}, \end{aligned} \quad (0.3.7)$$

where  $\delta$  is the phase delay between stress and particle velocity. The substitution of these expressions leads to a description of the response of the system in terms of a frequency-dependent effective viscosity:

$$\frac{\mathbf{T}}{\partial \xi / \partial x} = \frac{\mathbf{T}_0 e^{j\delta}}{\partial \Xi / \partial x} = \frac{\eta^0}{1 + j\omega\tau_v} \equiv \eta^*(j\omega). \quad (0.3.8)$$

The effective viscosity obtained here is a complex number and contains a contribution [the imaginary part of  $\eta^*(j\omega)$ ] which implies that such a medium has the property of a dynamic shear modulus  $G^*(j\omega) = G' + jG''$  with a nonzero real part. The real part of the complex viscosity coefficient at any frequency decreases uniformly from the low-frequency value  $\eta^0$  to zero as the frequency increases.

$$\eta'(\omega) = \eta^0 / [1 + (\omega\tau_v)^2]. \quad (0.3.9)$$

It is appropriate to define the equivalent complex shear modulus  $G^*(j\omega) \equiv G'(\omega) + jG''(\omega)$  by

$$G^*(j\omega) = \frac{\mathbf{T}_0 e^{j\delta}}{\partial \Xi / \partial x} = \frac{j\omega\eta^0}{1 + j\omega\tau_v} = \frac{G^\infty(\omega\tau_v)^2}{1 + (\omega\tau_v)^2} + j \frac{G^\infty\omega\tau}{1 + (\omega\tau_v)^2}, \quad (0.3.10)$$

where  $G^\infty \equiv \eta^0/\tau_v$ . The real part of the complex shear modulus (shown in Fig. 2b of Part 3) varies from zero at zero frequency to an asymptotic value  $G^\infty$  at frequencies very much greater than  $f_v = 1/2\pi\tau_v$ . The frequency  $f_v$  is thus called the *relaxation frequency* for the viscous mechanism. The imaginary part of the complex shear modulus increases from zero at zero frequency to a maximum at the relaxation frequency and falls again to zero at indefinitely high frequencies. As the frequency increases, the liquid exhibits an effective nonzero real part of the shear modulus and this property allows the propagation of heavily damped shear waves to occur. The propagation velocity as well as the absorption coefficient will be strongly dependent on frequency:

$$v_s = v_s^0 \frac{[1 + (G''/G')^2]^{1/4}}{\cos(G''/2G')}, \quad \alpha_s = \frac{2\pi f}{v_s^0} = \frac{\sin(G''/2G')}{[1 + (G''/G')^2]^{1/4}}, \quad (0.3.11)$$

where  $v_s^0 = (G'/\rho_0)^{1/2}$ , the limiting shear wave speed as the frequency approaches zero. We therefore observe a drastic change in the behavior of a liquid medium of moderate viscosity as the frequency of an applied acoustic perturbation is varied. At low frequencies it behaves like a viscous liquid that does not allow the propagation of shear waves. At frequencies considerably higher than the relaxation frequency, the equations predict that the absorption coefficient due to viscosity should approach a constant nonzero value and that the velocity of propagation of such waves should approach a constant value which is determined by  $G^\infty$ . It is found in the case of some hydrocarbon oils<sup>9</sup> that  $G^\infty$  is of the order of magnitude  $10^{10}$  dyn/cm<sup>2</sup>, i.e., within two orders of magnitude of the values characteristic of metals. In other words, the liquid is behaving much like a glass. The first known example of materials exhibiting behavior that is determined predominately by a single viscous relaxation mechanism of the type just described was molten zinc chloride.<sup>10</sup> Many other liquids behave as if several such viscous relaxation processes, described by different relaxation times and magnitudes, were superposed, as described in Part 3.

As one might expect, the observed behavior of most materials, including those of biological interest, is by no means so simple that it can be described adequately by a single relaxation time. It is found that sonic parameter magnitudes generally vary less drastically with frequency than the predictions of a single relaxation process require. Such behavior can be encompassed within the theory of relaxation processes by supposing that a discrete number of such processes are operative at the same frequency (each process may be described by a different value of the relaxation time) or, alternatively, by supposing that a continuous distribution of relaxation times exists. Since the latter possibility is more general, attention here will be confined to it. For the viscosity relaxation mechanism, Eq. (0.3.10) would be replaced by

$$G^*(j\omega) = G^\infty \int_{\tau_a}^{\tau_b} g_1(\tau) \frac{(\omega\tau)^2 d\tau}{1 + (\omega\tau)^2} + jG^\infty \int_{\tau_a}^{\tau_b} g_1(\tau) \frac{\omega\tau d\tau}{1 + (\omega\tau)^2}, \quad (0.3.12)$$

where  $\tau_a$  and  $\tau_b$  are bounds of the distribution of relaxation times. The distribution function  $g_1(\tau)$ , bounded by the values zero and one, expresses the contribution to the complex shear modulus which is derived from pro-

<sup>9</sup> A. J. Barlow and J. Lamb, *Proc. R. Soc. London Ser. A* **253**, 52 (1959).

<sup>10</sup> G. Gruber and T. A. Litovitz, *J. Chem. Phys.* **40**, 13 (1964).



cesses having relaxation times between  $\tau$  and  $\tau + d\tau$ . These concepts are considered in detail by Harrison and Barlow in Part 3.

Equations (0.3.4) and (0.3.9) may be combined and represented by

$$\alpha_v v / f = \alpha_v \lambda = 2(\alpha_v \lambda)_{\max} \omega \tau_v / [1 + (\omega \tau_v)^2], \quad (0.3.13)$$

where  $\eta'$  is identified with  $\eta$  and  $(\alpha_v \lambda)_{\max} = 4\pi^2 \eta^0 f_v / 3\rho_0 v_l^2$  is the maximum value of the relaxational absorption in unit wavelength, attained at the relaxation frequency  $f_v = 1/2\pi\tau_v$ , i.e., when  $f/f_v = \omega\tau = 1$  or  $\log(\omega\tau) = 0$ . An alternative formulation of Eq. (0.3.13) is

$$\alpha_v / f^2 = B / [1 + (f/f_v)^2], \quad (0.3.14)$$

where  $B = 8\pi^2 \eta / 3\rho_0 v_l^3 = 2(\alpha_v \lambda)_{\max} / v f_v$ .

The relaxational concept can now be extended to include processes that respond with a time delay to local changes of pressure or temperature. Examples of such processes are structural changes responding to pressure and redistribution of energy among vibrational degrees of freedom at the molecular level responding to temperature. Any process involving a change of molar volume or molar enthalpy will respond to the perturbations of local pressure [as given by Eq. (0.1.10)] or local temperature [as given by Eq. (0.1.12)]. If the reactants and products of the process are initially present in approximately equal mole fractions (i.e., if the process is approximately in equilibrium), then its contribution to the absorption of ultrasonic energy can be significant and expressions (0.3.13) and (0.3.14) apply with values of  $A$ ,  $(\alpha_r \lambda)_{\max}$ , and  $\tau$  governed by the process

$$\alpha_r v / f = \alpha_r \lambda = 2(\alpha_r \lambda)_{\max} \omega \tau / [1 + (\omega \tau)^2] \quad (0.3.15)$$

$$\alpha_r / f^2 = A / [1 + (f/f_r)^2], \quad (0.3.16)$$

where  $\alpha_r$ ,  $\tau$ , and  $A$  are generalizations of  $\alpha_v$ ,  $\tau_v$ , and  $B$ , respectively. Consequently, ultrasonic absorption measurements as a function of frequency offer a technique for exploring the kinetics of certain fast physicochemical reactions. In general, both volumetric and enthalpic changes accompany a reaction and complexities arise early in the interpretation of such measurements. The subject is discussed in detail by Slutsky in Part 4.

In liquids of low viscosity the relaxation times  $\tau = 1/2\pi f_r$  for the volumetric and enthalpic processes are frequently several orders of magnitude greater than  $\tau_v$  for the viscosity relaxation process. Therefore, the viscous process contributes the frequency-independent amount  $B$  to the quantity  $\alpha/f^2$  in the range of interest for studying many physicochemical reactions.

Three options for representing such typical relaxational behavior of

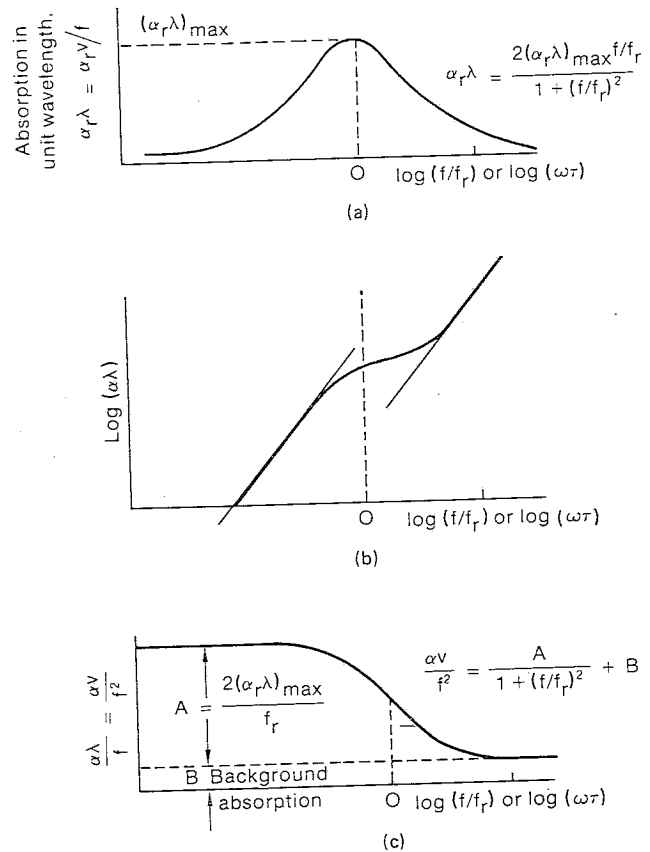


FIG. 1. (a) Relaxational contribution to the ultrasonic absorption per wavelength  $\alpha_r \lambda$  for a single relaxation process. (b) Absorption in unit wavelength  $\alpha_r \lambda$  for a single relaxation process added to absorption due to viscosity  $\alpha_v = B f^2$ . (c) Absorption parameter  $\alpha_r / f^2$  for a single relaxation process and background absorption due to viscosity.

media are shown in Fig. 1. Figure 1a in analogy with Fig. 2b of Part 3 shows a peak in the absorption in unit wavelength for a general relaxation process other than viscosity (which would give rise to another peak at much higher frequencies). Figure 1b shows the same quantities with the ordinate plotted on a logarithmic scale. A relaxation process causes the curve to transfer from one line of slope 2 to another displaced to higher frequencies. Figure 1c shows the measured value of the absorption parameter  $\alpha_r / f^2$  where the viscosity contributes the background absorption  $B$  (which tends to zero only at much higher frequencies):

$$\left(\frac{\alpha}{f^2}\right)_{\text{meas}} = \frac{A}{1 + (f/f_r)^2} + B. \quad (0.3.17)$$

The occurrence of relaxation is shown by the sigmoid form of  $\alpha/f^2$  plotted against frequency. Two measurements of  $\alpha/f^2$  (for example, at the highest and lowest frequencies available) are sufficient to show whether at least one relaxation region lies between the frequencies employed. Methods of measuring  $\alpha$  are discussed in detail by Breazeale, Cantrell, and Heyman in Part 2.

The occurrence of relaxation is not only associated with a peak in the relation between the absorption in unit wavelength and the frequency but it is also accompanied by dispersion of the sound speed in the same frequency range, i.e.,

$$v^2 = (v^0)^2 + 2v^0 \Delta v \frac{\omega\tau}{1 + (\omega\tau)^2}, \quad (0.3.18)$$

where  $v^0$  is the limiting sound speed as frequency tends to zero. The amplitude of the absorption peak and the increment of sound dispersion  $\Delta v$  are related as

$$\Delta v/v \approx (\alpha_r\lambda)_{\text{max}}/\pi, \quad (0.3.19)$$

provided  $(\alpha_r\lambda)_{\text{max}}/\pi \ll 1$ , which is the case for *compressional* waves in most materials of interest.

Since the magnitude of the velocity dispersion is very small (usually less than 1%), very accurate measurement of velocity is required if quantitative deductions are to be made. Observation of velocity dispersion in connection with an absorption maximum provides valuable confirmation that relaxational behavior is being observed. If only part of the relaxational spectrum is accessible experimentally, the following equation provides a useful relationship between  $\alpha_r\lambda$  and the slope of the velocity dispersion curve at the same frequency:

$$\frac{\alpha_r\lambda}{dv/df} = \frac{1}{2v\tau} (1 + \omega\tau). \quad (0.3.20)$$

#### 0.4. Attenuation

Any process that removes energy from a traveling acoustic wave but does not dissipate that energy as heat contributes to the attenuation of the acoustic wave but not the absorption by the medium. The primary contribution to attenuation in addition to absorption is acoustic scattering by inhomogeneities in the medium.

Scattering of ultrasound is governed by the same principles as scattering of electromagnetic radiation, e.g., radar signals in the atmosphere or light in a turbid liquid. Three cases are distinguished, depending on the ratio of the wavelength of the radiation to the linear dimension of the inhomogeneities in "refractive index"; i.e., much less than unity, much greater than unity, or comparable in magnitude. When the ratio is much less than unity, behavior is approximately described by the theory for reflection and refraction at plane interfaces. When the ratio is much greater than unity (small scatterers), Rayleigh's theory is applicable. The greatest complexity arises when wavelength and linear dimension of the inhomogeneities are comparable in magnitude.

In all three cases, however, the acoustic problem is inherently more complicated than the electromagnetic problem, because elastic media support both bulk compressional and shear waves, in general, and surface waves can be generated at interfaces. Furthermore the acoustic "refractive index" will vary in response to variations of either density or elastic modulus or both. Further difficulties arise when the absorption by the medium is not negligible and when the locations and properties of the inhomogeneities are time dependent. It is therefore hardly surprising that the theory of scattering of ultrasound is in an early stage of development.

Scattering in polycrystalline media is discussed in detail by Papadakis in Part 5. Scattering by noncrystalline media, such as biological tissues, is a subject of ongoing research on which a consensus is not yet available. Readers seeking information on this subject may consult original articles by Sigelmann and Reid,<sup>11</sup> Shung *et al.*,<sup>12</sup> and Waag *et al.*<sup>13</sup> and a review by Chivers.<sup>14</sup> Texts by Morse and Ingard<sup>15</sup> and Chernow<sup>16</sup> provide theoretical bases for the two major aspects of the problem, i.e., scattering from discrete entities or in inhomogeneous continua. Twersky's<sup>17</sup> work provides a foundation for studies of multiple scattering.

<sup>11</sup> R. A. Sigelmann and J. M. Reid, *J. Acoust. Soc. Am.* **53**, 1351 (1973).

<sup>12</sup> K. K. Shung, R. A. Sigelmann, and J. M. Reid, *IEEE Trans. Biomed. Eng.* **BME-24**, 460 (1976).

<sup>13</sup> R. C. Waag, R. M. Lerner, and R. Gramiak, "Seminar on Tissue Characterization," NBS Special Publication 453, pp. 213-228, U.S. Government Printing Office, Washington, D.C., 1976.

<sup>14</sup> R. C. Chivers, *Ultrasd. Med. Biol.* **3**, 1 (1977).

<sup>15</sup> P. M. Morse and K. U. Ingard, "Theoretical Acoustics," McGraw-Hill, New York, 1968.

<sup>16</sup> L. A. Chernow, "Wave Propagation in a Random Medium," Dover, New York, 1960.

<sup>17</sup> V. Twersky, *J. Res. Nat. Bur. Stand.* **64D**, 715 (1960).