A Primary Method for the Determination of Ultrasonic Intensity with the Elastic Sphere Radiometer *

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Summary

An evaluation is carried out of the acoustic radiation force function $Y = F_r c/(\pi a^2 I)$, where F_r is the measurable radiation force on a sphere, c is the velocity of sound in the suspending medium, a is the radius of the suspended sphere, and I is the intensity of the plane progressive sound field incident upon the sphere. Comparison is made of the inelastic and elastic cases and experimentally verified for commercially available grade 10 type 440C stainless steel ball bearings. It is concluded that for observations avoiding the resonance minima of Y, acoustic intensity can be determined with an absolute accuracy of about $\pm 3\%$.

Eine elementare Methode zur Bestimmung der Ultraschallintensität mit Hilfe des elastischen Kugel-Radiometers

Zusammenfassung

Es wird eine Auswertung der akustischen Strahlungskraft-Funktion $Y = F_r c/(\pi a^2 I)$ vorgenommen. Dabei ist F_r die meßbare Strahlungskraft auf der Kugel, c die Schallgeschwindigkeit im Medium, a der Radius der vom Medium umgebenen Kugel und I die Intensität des auf die Kugel auftreffenden, ebenen, fortschreitenden Schallfeldes. Die inelastischen und die elastischen Fälle werden verglichen und für im Handel erhältliche, rostfreie Kugellager (Grad 10, Typ 440C) experimentell verifiziert. Für Beobachtungen unter Umgehung der Resonanz-Minima von Y kann die Schallintensität mit einer absoluten Genauigkeit von ungefähr $\pm 3\%$ bestimmt werden.

Méthode primaire pour déterminer l'intensité d'ultrasons au moyen du radiomètre à sphère élastique

Sommaire

On calcule la force de pression de radiation $Y = F_r c/(\pi a^2 I)$, où F_r est la force moyenne de radiation sur une sphère, telle qu'elle est mesurable; c est la vitesse du son dans le milieu où la sphère est plongée, a le rayon de la sphère, I l'intensité du champ sonore plan progressif incident sur la sphère. On compare entre eux les cas élastique et non élastique, et l'on vérifie les prévisions avec pour sphère une bille de roulement en acier inoxydable du type 10-440C que l'on trouve dans le commerce. On trouve que si l'on évite les minima de résonance de Y, l'intensité acoustique peut être mesurée avec une erreur absolue d'environ $\pm 3\%$.

1. Introduction

The radiation force exerted by a plane ultrasonic beam on a sphere provides a convenient, accurate, and primary method for determining the acoustic intensity of that field [1]. Both spatial-average and nearly-point values can be obtained, depending upon the relative values of the beam and sphere dimensions. The simplicity of the method and the increasing need for calibration of ultrasonic transmitters, receivers, and standards for the effusion of applications in industry and medicine account for its appeal.

2. Results and discussion

Briefly, the displacement d of a freely suspended sphere, as shown in Fig. 1, is related to the radiation force F_r as

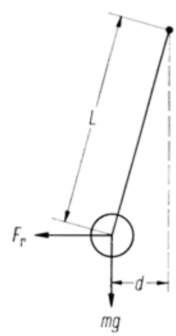

Fig. 1. Forces acting upon a sphere suspended in a sound field.

$$F_r = \frac{m g d}{(L^2 - d^2)^{1/2}}, \tag{1}$$

where L is the suspension length, g the acceleration due to gravity, and m is the mass of the sphere plus

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the mass of that fraction of the suspension structure involved, both corrected for bouyancy. The suspension is generally bifilar to minimize lateral displacements of the sphere and may be a very small gauge nylon monofilament. Thus F_r is readily determined experimentally and has been related to the acoustic intensity I of the ultrasonic field of interest by

$$I = \frac{F_r c}{\pi a^2 Y} \tag{2}$$

where a is the radius of the sphere, c is the speed of ultrasound in the fluid medium and the dimensionless constant Y, the radiation force per unit cross section per unit energy density, is known as the acoustic radiation force function.

King [2] evaluated Y for inelastic spheres. More recently, Hasegawa and Yosioka [3] derived Y for the case where the material of the sphere is described by two elastic constants. The acoustic radiation force function was found to differ considerably when the elasticity of the sphere is included in the Y formulation. For the inelastic case Y increases monotonically rather steeply, as a function of ka, to $ka \simeq 2$ after which it approaches unity asymptotically, where k is the wave number $(2\pi/\lambda)$. The effect of including two elastic constants is shown in Fig. 2 where resonance phenomena are manifested as a sequence of pronounced maxima and minima in the Y vs. ka function. Comparison of the elastic and inelastic cases shows that the Y functions are similar to approximately $ka = 2$, with the inelastic Y function nearly superposing the elastic Y function in the region $ka = 2$ to 4. For ka greater than 8, the inelastic Y function remains greater as the elastic Y function does not exceed 0.946.

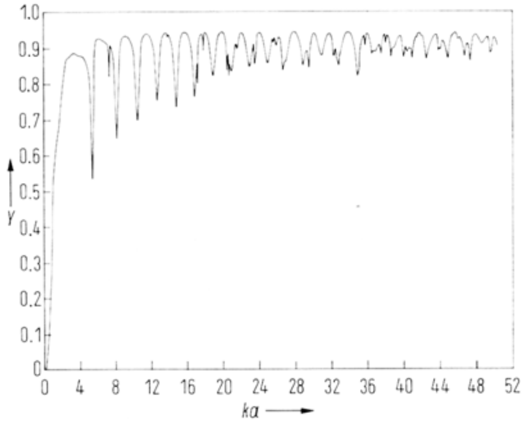

Fig. 2. The acoustic radiation force function , Y, as a function of ka for type 440C stainless steel. See Table I for physical parameters.

It was considered creditable to examine this sound field measurement method in view of the more appropriate theoretical framework provided by Hasegawa and Yosioka [3]. Thus, a computer program was prepared to evaluate the acoustic radiation force function according to their formulae which included the inelastic case by allowing for the use of very high values of the speed of sound, viz., of the order of 10^9 cm/s. The latter exhibited agreement with the computations of Fox [4], who first employed King's theory [2] for absolute determination of acoustic intensity by means of radiation force measurement. Fig. 2 shows the acoustic radiation force function, Y, computed in ka steps of 0.04 to $ka = 20$ and in ka steps of 0.08 to $ka = 50$ for grade 10 440 C stainless steel (Winsted Precision Ball, Winsted, Conn. USA). Table I lists the appropriate physical constants of this material, used routinely for sound field measurements in this laboratory. Table II lists the values of Y as a function of ka in increments of 0.1 ka, and Table III lists the extrema of Y for ka values 0 to 20.

In order to asses the possible errors introduced into the measurement method by, for example, temperature variations, computations were carried out for the stainless steel material used by Hasegawa and Yosioka [3] in water at 55 °C and 2 °C. The physical properties of this stainless steel are also listed in Table I. Since the temperature coefficient of water is considerably greater than that of steel, only the parameters of the fluid were altered

Table I.
Physical constants for Y calculations.

Material	Density g/cm³	Poisson's Ratio	Compressional Velocity km/s	Shear Velocity km/s
Stainless steel (Hasegawa & Yosioka)	7.90	0.264	5.240	2.978
Stainless steel Type 440C	7.84	0.2962	5.854	3.150
Water	1.0		1.50	

Table II.
Values of the elastic acoustic radiation force function, Y, as a function of ka in increments of 0.1 ka for stainless steel 440C. See Table I for physical parameters.

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.000	0.000	0.002	0.008	0.022	0.049	0.092	0.151	0.222	0.302
1	0.384	0.459	0.523	0.574	0.610	0.635	0.655	0.673	0.693	0.718
2	0.746	0.776	0.805	0.829	0.848	0.861	0.868	0.872	0.874	0.876
3	0.879	0.881	0.884	0.886	0.887	0.887	0.885	0.884	0.882	0.881
4	0.880	0.880	0.879	0.879	0.877	0.874	0.871	0.866	0.860	0.852
5	0.841	0.823	0.793	0.737	0.631	0.534	0.700	0.851	0.902	0.919
6	0.925	0.927	0.927	0.927	0.926	0.925	0.923	0.922	0.920	0.918
7	0.916	0.913	0.908	0.896	0.770	0.907	0.900	0.885	0.867	0.839
8	0.792	0.717	0.649	0.700	0.808	0.873	0.904	0.920	0.927	0.931
9	0.933	0.934	0.934	0.933	0.931	0.929	0.925	0.921	0.915	0.906
10	0.894	0.874	0.843	0.796	0.742	0.703	0.716	0.789	0.855	0.894
11	0.916	0.928	0.934	0.938	0.940	0.941	0.940	0.939	0.936	0.933
12	0.928	0.912	0.910	0.895	0.872	0.837	0.791	0.756	0.770	0.822
13	0.870	0.901	0.920	0.930	0.937	0.940	0.942	0.942	0.939	0.941
14	0.937	0.932	0.925	0.915	0.900	0.878	0.846	0.803	0.754	0.756
15	0.826	0.871	0.901	0.920	0.932	0.938	0.942	0.944	0.945	0.944
16	0.942	0.939	0.933	0.926	0.914	0.898	0.874	0.840	0.793	0.766
17	0.803	0.843	0.837	0.903	0.922	0.932	0.938	0.939	0.932	0.911
18	0.945	0.943	0.939	0.933	0.924	0.912	0.894	0.871	0.845	0.826
19	0.828	0.850	0.879	0.902	0.919	0.931	0.938	0.942	0.944	0.945
20	0.944	0.942	0.937	0.929	0.912	0.828	0.887	0.858	0.854	0.839
21	0.836	0.851	0.873	0.894	0.907	0.910	0.901	0.905	0.927	0.938
22	0.942	0.942	0.940	0.935	0.928	0.919	0.905	0.888	0.868	0.852
23	0.847	0.858	0.877	0.895	0.908	0.909	0.887	0.880	0.927	0.939
24	0.943	0.943	0.941	0.936	0.930	0.920	0.908	0.892	0.875	0.861

Table III.
Y-function extrema for stainless steel 440C in the ka range $0 \cdots 20$.

ka	Y
3.435	0.887
5.500	0.534
6.199	0.927
7.400	0.770
7.456	0.910
8.210	0.648
9.132	0.934
10.556	0.699
11.519	0.941
12.734	0.754
13.655	0.942
13.865	0.932
13.900	0.941
14.870	0.735
15.792	0.945
16.881	0.764
17.174	0.856
17.216	0.800
17.677	0.939
17.886	0.892
18.012	0.945
18.900	0.826
19.897	0.845

in making the calculations. Fig. 3 shows the results wherein it is seen that the first minimum moves from $ka = 5.25$ to $ka = 5.70$ in the temperature range treated. At larger ka values, where the maxima and minima became relatively more closely spaced, it is clear that exceptional temperature regulation and accurate values for the velocities of sound

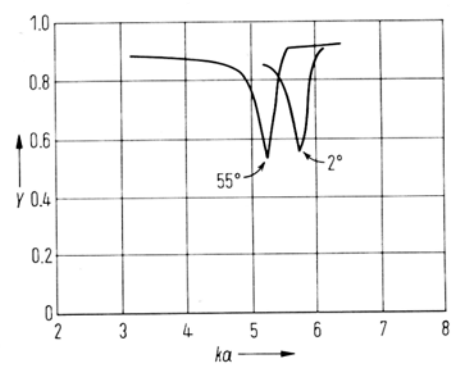

Fig. 3. The influence of temperature variations on the acoustic radiation force function for the stainless steel material used by Hasegawa and Yosioka. See Table I for physical parameters.

are essential for precision acoustic intensity measurements with this method.

Experimental data, with type 440 C stainless steel spheres, were obtained to compare theory with experiment. Here, five ball bearings whose diameters varied from 1/16 in. to 1/8 in. in steps of 1/64 in. were utilized at the three ultrasonic frequencies of 0.916 MHz, 0.940 MHz, and 1.00 MHz to produce the data shown in Fig. 4. The spheres

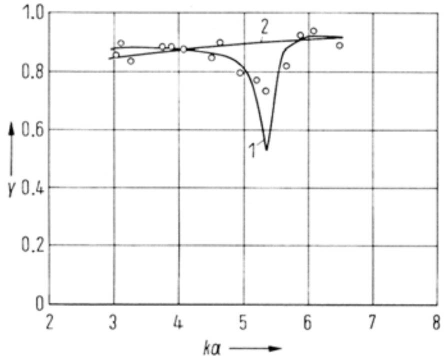

Fig. 4. Comparison of theory and experiment in determination of the acoustic radiation force function Y.
Curve 1: elastic case, type 440C stainless steel,
curve 2: inelastic case. See text for explanation of plotted points.

were suspended approximately 10 cm with 0.002 in diameter nylon monofilment in a bifilar arrangement. In order to eliminate radiation force related errors due to unavoidable masses of adhesive materials on the spherical surface, at a point of contact of the suspending fiber, diametrical holes, of dimensions equal to the nylon filament, were drilled by an electron beam process. The filament may then be secured to the sphere either by simply passing through the hole, which aligns the hole horizontally (to the earth's surface) and requires no additional adhesive, or by apical insertion, which aligns the hole vertically and requires a small quantity of adhesive. The adhesive used for the latter arrangement may have acoustic properties very similar to that of the suspending filament. Both securing methods have been employed with no discernible differences, though the data shown in Fig. 4 were obtained with the apical insertion method.

The two curves of Fig. 4 relate to computations from the elastic, Hasegawa and Yosioka [3] theory and the inelastic King [2] theory, respectively, and the plotted points are the relative magnitudes of the experimentally determined acoustic radiation force function normalized to the elastic curve at $ka=4$. The latter value was chosen for the normalization process because the two theories yield practically identical values here. In the flat portions of the curves, viz., for ka approximately 3 to 4 and 5.75 to 6.25, the repeatability of the Y factor appeared within 5% peak to peak. It is seen that a clear departure of the experimental data from the inelastic curve appears in the neighborhood of ka value where the elastic curve exhibits a minimum at 5.50. However, the minimum value obtained does not quite reach that predicted by the elastic theory. Experimentally it cannot be stated that the true minimum has been reached, i.e., the frequencies and spheres used may not have provided the proper ka minimum value. Additional explanations may be that the ka value at which the minimum occurs is slightly above or below the ka value of 5.50, that the drilled hole may have altered the resonant characteristics of the sphere, and that the amplitude of the sound field was not as uniform over the diameter of the sphere as the two theories require.

Nevertheless, it is possible to conclude that the ultrasonic intensity can be determined with an acceptable degree of accuracy with the elastic sphere radiometer. However, the Hasegawa and Yosioka [3] formulae should be used to obtain the acoustic radiation force function. When the minima in the Y factor can be avoided, and measurements confined to the more flat portions, the accuracy of the absolute determination of intensity is about $\pm 3\%$. The recent work by Hasegawa and Yosioka [5] with fused silica spheres suggests that greater degrees of accuracy may be obtained when appropriate selection of the mechanical properties of the spheres is possible.

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