

loading of the surface, present theory should still be adequate and the equivalent acoustic roughness levels should be smaller than the physical by a factor of roughly 100. The experimental evidence suggests that this may be true only for frequencies below approximately 20 Hz.

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13.2, 13.3; 11.2

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## Model Sound Channel with Sinusoidal Rays

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Material is herein reproduced of an Admiralty Research Laboratory note in which it was shown that, if the square of the reciprocal of the sound velocity is a quadratic function of depth (a type of profile due, it is now pointed out, to M. A. Pedersen), then all the ray paths in the neighborhood of the sound-velocity minimum take the form of sine curves. The space and time periods are calculated.

IN AN EXTENSIVE DISCUSSION,<sup>1</sup> STEWART HAS QUOTED A REPORT<sup>2</sup> with the above title, whose date appears to give it priority over work by Pedersen.<sup>3</sup> Soon after the issue of my report, however, I learned that Pedersen and his colleagues had already made extensive use of the type of formula I was studying. My report was very short, and I reproduce it in full below, while making it clear that any priority belongs to Pedersen.

It is well known<sup>4</sup> that sound rays traveling at a small-enough inclination to the horizontal in the neighborhood of a minimum of the sound velocity in the ocean cycle about the minimum, remaining confined to a "sound channel." It is usual to calculate the paths of such rays by approximating to the law of dependence of velocity on depth by a series of linear segments, for each of which the ray paths take the form of arcs of circles. This is not a very convenient approximation near a turning value of the sound velocity (and there may be doubts of the validity of some of the results obtained from it). The question arises whether a form of approximation to the velocity-depth law can be found that gives, for the sound rays, oscillating curves of simple analytical properties. It is shown here that, if the reciprocal of the square of the sound velocity is a quadratic function of depth—which is a convenient approximation in the neighborhood of the minimum—then all the rays take the form of sine curves whose parameters are readily calculated.

The sound velocity  $v$  is supposed to depend only on the depth  $z$  and to be independent of the range  $r$  and of bearing. The differential equations of the rays can then be deduced directly from Snell's law, giving

$$dr/ds = kv, \quad (1)$$

where  $s$  represents path length and  $k$  is constant along each ray. Since  $ds^2 = dr^2 + dz^2$ , it follows at once that

$$dz/ds = (1 - k^2v^2)^{1/2}, \quad (2)$$

$$dr/dz = kv(1 - k^2v^2)^{-1/2} \quad (3)$$

and

$$= k(\omega^2 - k^2)^{-1/2}, \quad (4)$$

where  $\omega = 1/v$ .

Now, suppose that  $\omega^2$  is a quadratic function of  $z$ , i.e.,

$$\omega^2 = a^2 - b^2(z - z_0)^2, \quad (5)$$

giving a minimum of  $v$  where  $z = z_0$ . Then,

$$dr/dz = k\{a^2 - k^2 - b^2(z - z_0)^2\}^{-1/2}, \quad (6)$$

an equation that can be integrated immediately to give

$$b(z - z_0) = (a^2 - k^2)^{1/2} \sin[b(r - r_0)/k], \quad (7)$$

representing a sine curve passing through the sound-channel axis  $z = z_0$  at the arbitrary range  $r_0$  at the arbitrary inclination  $\cos^{-1}k/a$  to the horizontal. The spatial period is  $2\pi k/b$ , and the vertical amplitude is  $(a^2 - k^2)^{1/2}/b$ .

Again, Eq. 1 gives

$$dr/dt = kv^2, \quad dt/dr = \omega^2/k; \quad (8)$$

and, since

$$\begin{aligned} \omega^2 &= a^2 \cos^2\{b(r - r_0)/k\} + k^2 \sin^2\{b(r - r_0)/k\} \\ &= \frac{1}{2}(a^2 + k^2) + \frac{1}{2}(a^2 - k^2) \cos\{2b(r - r_0)/k\}, \end{aligned} \quad (9)$$

the travel time is given by

$$t = \frac{a^2 + k^2 r}{2k} - \frac{a^2 - k^2}{4b} \left( \sin \frac{2b(r - r_0)}{k} + \sin \frac{2br_0}{k} \right). \quad (10)$$

The time taken to travel a cycle is, thus,

$$T = \pi(a^2 + k^2)/b. \quad (11)$$

This concludes the text of ARL/N48/L.

I may remark that it is also possible to generalize the law of the sound-velocity profile to make  $c^{-2}$  a cubic or quartic function of depth and still make use of known functions (elliptic functions). The use of a cubic can give some asymmetry about the turning point and leads to Weierstrassian elliptic functions. The use of a quartic, without linear or cubic terms, can represent a sharpened or flattened curve and gives rise to Jacobian elliptic functions. However, it does not appear that, even in these two simple cases, the form in which the functions are usually tabulated is directly suited to this problem, so that the gain to be obtained from using known analytic functions is questionable.

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16.4

## Ultrasonic Irradiation of Enzyme Solutions

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It is shown that inactivation of enzymes in solution by 3-MHz ultrasound in the intensity range 1-3 W/cm<sup>2</sup>, which has been reported by other investigators, is, in fact, due to the presence of rubber materials used as part of the containing vessel during the experiments.

THE NONTHERMAL MECHANISM(S) OF INTERACTION OF ULTRASOUND and biological media is poorly understood. The inactivation of enzyme solutions by cavitating ultrasound has been reported on

numerous occasions.<sup>1</sup> Interactions in the absence of cavitation are known to occur in tissues<sup>2,3</sup> and DNA solutions.<sup>4</sup> Recently,<sup>5</sup> an extensive study has been completed in which five enzymes ( $\alpha$ -chymotrypsin, trypsin, ribonuclease, aldolase, and lactate dehydrogenase), each catalyzing a different reaction, were treated under cavitating and noncavitating ultrasonic conditions in the frequency range 20 kc/sec 30 MHz, intensity range 0.5–10<sup>3</sup> W/cm<sup>2</sup> (depending upon frequency), and for irradiation times from 10<sup>-1</sup>–10<sup>2</sup> sec (depending upon intensity). Temperature, pH, and concentration of the solutions depended upon the properties of the enzyme and, where appropriate, were varied. Analytical procedures included measurements of enzyme activity, specific optical rotation, uv absorption spectra and the sedimentation coefficient. Chromatographic analyses were also performed. The results of these studies show clearly that, under the conditions of these experiments, the presence of cavitation is necessary for ultrasound to affect enzyme solutions *in vitro*.

Stefanović *et al.*<sup>6</sup> and Stefanović *et al.*<sup>7</sup> have reported inactivation of trypsin and diastase solutions, respectively, with 3-MHz ultrasound at intensities in the range 1.5–2.5 W/cm<sup>2</sup>. The samples treated were contained in a glass cylinder closed at one end by a rubber membrane. Cavitation can be eliminated as the mechanism since it is well established that the cavitation threshold in, for example, aerated water at 3 MHz is greater than 10<sup>3</sup> W/cm<sup>2</sup>.<sup>8</sup> Thermal mechanisms can be eliminated by the manner in which the experiments were conducted.<sup>6,7</sup> In attempts to explain these results, which are at variance with those indicated above,<sup>5</sup> experiments were conducted in this laboratory in which  $\alpha$ -chymotrypsin was irradiated with 1-MHz ultrasound at 75 W/cm<sup>2</sup> for varying periods of time, with the solution contained in a rubber bag. Results similar to those reported in Refs. 6 and 7 were obtained. In the experiments reported in Ref. 5, the solutions were always in contact with materials chemically inert to these aqueous solutions, *viz.*, saran, plexiglass, glass, etc. Subsequent experiments in which the rubber bag material was macerated by a Waring blender in the presence of the enzyme solution also resulted in reduction of enzymatic activity, while employment of the blender with the enzyme solution and no rubber material present showed no change in activity. The uv absorption spectra of solutions that were agitated, either by ultrasonic irradiation or in the Waring blender, with the rubber material present, showed more than a 200% increase in absorption over the entire wavelength range 240–320  $\mu$ . Further, simply filling the rubber container with distilled water and irradiating with ultrasound under the same conditions

as for the enzyme solutions resulted in the appearance in the distilled water of the same substance that absorbs strongly in ultraviolet. These changes in the uv-absorption spectrum were highly reproducible from sample to sample. Control samples that were placed in rubber containers but not irradiated showed no change either in activity or in their uv-absorption spectra. Hence, whatever the nature of the substance that appears within the container, it is produced by mechanical agitation and thus, it must be concluded that the results reported in Refs. 6 and 7 are not due directly to an interaction between the ultrasonic waves and the protein molecules in solution, but rather to an unspecified reaction between the solution and the rubber used for the container.

**Acknowledgment:** This work was supported by the National Science Foundation and the Office of Naval Research, Acoustics Programs.

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## Erratum—Table of Contents Vol. 40, No. 4, October 1966

IN THE OCTOBER ISSUE, THE FOLLOWING LETTER TO THE Editor was omitted from the Table of Contents—

### Pitch Shifts of Tones in Wide-Band Noise 912–913

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