loading of the surface, present theory should still be adequate and
the equivalent acoustic roughness levels should be smaller than
the physical by a factor of roughly 100. The experimental evidence
suggests that this may be true only for frequencies below approxi-
mately 20 Hz.

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the U. S. Navy Electronics Laboratory for furnishing the underice
profile.

* Present address: U. S. Navy Underwater Sound Lab., New London,
Conn. 06321.
1 H. W. Marsh and R. H. Mellon, "Underwater Sound Propagation in the
2 R. H. Mellen and H. W. Marsh, "Underwater Sound in the Arctic
Ocean," AAVCO Report to U. S. Navy Underwater Sound Laboratory (16
August 1965).
3 W. K. Lyon, "Ocean and Sea Ice Research in the Arctic Ocean via
4 R. W. Burling, "Wind Generation of Waves on Water," Doctoral
5 H. W. Marsh and R. H. Mellon, "Boundary Scattering Effects in
6 A. R. Milne, "Underwater Backscattering Strength of Arctic Pack
7 J. R. Brown, "Ray Acoustical Model of the Ocean Using a Depth
34, 601-603 (1963).
8 R. P. Chapman and H. D. Scott, "Backscatter Strength of Young Sea

Model Sound Channel with Sinusoidal Rays

A. G. D. WATSON

Admiralty Research Laboratory, Teddington, Middlesex, England

Material is herein reproduced of an Admiralty Research Laboratory note
in which it was shown that, if the square of the reciprocal of the
sound velocity is a quadratic function of depth (a type of profile due, it is now
pointed out, to M. A. Pedersen), then all the ray paths in the neighborhood
of the sound-velocity minimum take the form of sine curves. The space and
time periods are calculated.

In an extensive discussion,1 STEWART HAS QUOTED A REPORT2
with the above title, whose date appears to give it priority over
the work by Pedersen. Soon after the issue of my report, however,
I learned that Pedersen and his colleagues had already made
extensive use of the type of formula I was studying. My report
was very short, and I reproduce it in full below, while making
it clear that any priority belongs to Pedersen.

It is well known that sound rays traveling at a small-enough
inclination to the horizontal in the neighborhood of a minimum of
the sound velocity in the ocean cycle about the minimum, remain-
ing confined to a "sound channel." It is usual to calculate the
paths of such rays by approximating to the law of dependence of
velocity on depth by a series of linear segments, for each of which
the ray paths take the form of arcs of circles. This is not a very
convenient approximation near a turning value of the sound
velocity (and there may be doubts of the validity of some of the
results obtained from it). The question arises whether a form of
approximation to the velocity depth law can be found that gives,
for the sound rays, oscillating curves of simple analytical proper-
ties. It is shown here that, if the reciprocal of the square of the
sound velocity is a quadratic function of depth—which is a
common approximate approximation in the neighborhood of the
minimum—then all the rays take the form of sine curves whose parameters
are readily calculated.

The sound velocity v is supposed to depend only on the depth z
and to be independent of the range r and of bearing. The differ-
ential equations of the rays can then be deduced directly from
Snell's law, giving

\[ \frac{dr}{dz} = -kv \]  

where s represents path length and k is constant along each ray. Since
\( ds = dr + dz \), it follows at once that

\[ \frac{ds}{dz} = (1-kv^2)^{-1} \]

and

\[ w = (z^2-b^2)^{-1} \]

where \( \alpha = 1/r \).

Now, suppose that \( w \) is a quadratic function of \( z \), i.e.,

\[ w = z^2 - a^2(1-\beta^2)^2 \]

giving a minimum of \( w \) where \( z = z_0 \). Then,

\[ \frac{dr}{dz} = k\left(w - a^2\beta^2\right)^{-1} \]

an equation that can be integrated immediately to give

\[ b(z-z_0) = a^2(1-\beta^2)\sinh[(r-r_0)/b] \]

representing a sine curve passing through the sound-channel axis
\( z = z_0 \) at the arbitrary range \( r_0 \) at the arbitrary inclination \( \cos^{-1}k/a \)
to the horizontal. The spatial period is \( 2k/b \), and the vertical
amplitude is \( (a^2-b^2)/b \).

Again, Eq. 1 gives

\[ \frac{dr}{dz} = k\sinh[(r-r_0)/b] \]

and, since

\[ w = a^2\cos[(r-r_0)/b] + b^2\sinh[(r-r_0)/b] \]

the travel time is given by

\[ T = \sqrt{\frac{a^2+b^2}{2b}} \]

The time taken to travel a cycle is, thus,

\[ T = \pi(a^2+b^2)/b \]

This concludes the text of ARL/N48/L.

I may remark that it is also possible to generalize the law of the
sound-velocity profile to make \( w \) a cubic or quartic function of
depth and still make use of known functions (elliptic functions).
The use of a cubic can give some asymmetry about the turning
point and leads to Weierstrassian elliptic functions. The use of a
quartic, without linear or cubic terms, can represent a sharpened
or flattened curve and gives rise to Jacobian elliptic functions.
However, it does not appear that, even in these two simple cases,
the form in which the functions are usually tabulated is directly
suitable to this problem, so that the gain to be obtained from
using known analytic functions is questionable.

1 J. R. Brown, "Ray Acoustical Model of the Ocean Using a Depth
2 A. G. D. Watson, "Acoustic Sound Channel with Sinusoidal Rays,"
3 M. A. Pedersen, "Acoustic Intensity Anomalies Introduced by Cont-
4 M. Ewing and J. L. Worzel, "Long-Range Sound Transmission," Geol.

Received 13 June 1966

Ultrasonic Irradiation of Enzyme Solutions

R. M. MACLEOD AND FLOYD DUNN

Biophysical Research Laboratory, University of Illinois, Urbana, Illinois

It is shown that inactivation of enzymes in solution by 3-MHz ultrasonic
in the intensity range 1-7 W cm\(^{-2}\), which has been reported by other
investigators, is, in fact, due to the presence of rubber materials used as part
of the containing vessel during the experiments.

The NONTHERMAL MECHANISM(s) OF INTERACTION OF ULTRASONIC
and biological media is poorly understood. The inactivation
of enzyme solutions by cavitation ultrasound has been reported on

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Numerous occasions. Interactions in the absence of cavitation are known to occur in tissues and DNA solutions. Recently, an extensive study has been completed in which five enzymes (a-chymotrypsin, trypsin, ribonuclease, aldolase, and lactate dehydrogenase), each catalyzing a different reaction, were treated under cavitating and noncavitating ultrasound conditions in the frequency range 20 kc/sec to 30 MHz, intensity range 0.5 to 10\(^6\) W/cm\(^2\) (depending upon frequency), and for irradiation times from 10 to 10\(^{-6}\) sec (depending upon intensity). Temperature, pH, and concentration of the solutions depended upon the properties of the enzyme and, where appropriate, were varied. Analytical procedures included measurements of enzymic activity, specific optical rotation, uv absorption spectra and the sedimentation coefficient. Chromatographic analyses were also performed.

The results of these studies show clearly that, under the conditions of these experiments, the presence of cavitation is necessary for ultrasound to affect enzyme solutions in vitro.

Stefanović et al. and Stefanović et al. have reported inactivation of trypsin and diastase solutions, respectively, with 3-MHz ultrasound at intensities in the range 1.5 to 2.5 W/cm\(^2\). The samples treated were contained in a glass cylinder closed at one end by a rubber membrane. Cavitating can be eliminated as the mechanism since it is well established that the cavitation threshold in, for example, aerated water at 3 MHz is greater than 10\(^6\) W/cm\(^2\). Thermal mechanisms can be eliminated by the manner in which the experiments were conducted. In attempts to explain these results, those which are at variance with those indicated above, experiments were conducted in this laboratory in which a chymotrypsin was irradiated with 1-MHz ultrasound at 75 W/cm\(^2\) for varying periods of time, with the solution contained in a rubber bag. Results similar to those reported in Refs. 6 and 7 were obtained. In the experiments reported in Ref. 5, the solutions were always in contact with materials chemically inert to these aqueous solutions, viz., saran, plexiglass, glass, etc. Subsequent experiments in which the rubber bag material was macerated by a Waring blender with the solution present showed no change in activity. The uv absorption spectra of solutions that were agitated, either by ultrasonic irradiation or in the Waring blender, with the rubber material present, showed more than a 200% increase in absorption over the entire wavelength range 240 to 320 m\(\mu\). Further, simply filling the rubber bag container with distilled water and irradiating with ultrasound under the same conditions as for the enzyme solutions resulted in the appearance in the distilled water of the same substance that absorbs strongly in ultraviolet. These changes in the uv-absorption spectrum were highly reproducible from sample to sample. Control samples that were placed in rubber containers but not irradiated showed no change either in activity or in their uv-absorption spectra. Hence, whatever the nature of the substance that appears within the container, it is produced by mechanical agitation and thus, it must be concluded that the results reported in Refs. 6 and 7 are not due directly to an interaction between the ultrasonic waves and the protein molecules in solution, but rather to an unspecified reaction between the solution and the rubber used for the container.

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Erratum—Table of Contents
Vol. 40, No. 4, October 1966

In the October issue, the following Letter to the Editor was omitted from the Table of Contents—

Pitch Shifts of Tones in Wide-Band Noise 912-913
SYLVIA J. STEINER AND ARNOLD M. SMALL
University of Iowa, Iowa City, Iowa