In order to circumvent this limitation, a control system which can provide each section of DAVO with an individual set of temporal instructions is required. Each section would undergo a given change in its cross-sectional area at a different time and with a different rate, with the result that the vocal-tract analog would be capable of making almost any transition between two configurations. It is impractical to incorporate such a control system into the present programming device, however, and the problem of controlling DAVO by means of computer must be eventually considered.

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Ultrasonic Intensity Gain by Composite Transducers

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The radiation of plane acoustic waves into a medium by composite layered transducers composed of a piezoelectric element and two lossless coupling media is described analytically. The increase in acoustic intensity in the medium receiving the radiation produced by the composite structure over that produced by direct coupling (piezoelectric element in contact with the medium), for the same value of the electric field strength, is considered quantitatively and the results are presented graphically. Composite transducers of the type described can be expected to achieve considerably higher ultrasonic intensities (1000-fold increase) in both unfocused and focused beams than have heretofore been realized.

I. INTRODUCTION

THE production of ultrasonic radiation by composite transducers has received attention in the technical literature[1,2] and information on the propagation characteristics of sound through multilayered structures (including engineering design criteria) is to be found in almost all books on acoustics.[3] A general treatment of acoustic propagation through lossless structures of four or more layers has been published recently by Dianov,[4] who compares experimental results of radiated acoustic intensity and bandwidth for the direct radiated case with that incorporating two quarter-wave layers between the piezoelectric element and the radiation medium. The use of composite impedance transforming structures to couple vibrating piezoelectric elements to a medium has been discussed briefly by Fry and Dunn.[5] The technical literature is particularly lacking in information regarding optimum design conditions, i.e., values of the acoustic parameters and dimensions for the coupling media, for maximizing the acoustic output power per unit radiating area for a fixed value of the electric field strength.[6,7] An objective of this paper is the presentation of transducer design criteria for achieving increased power outputs.

High-intensity ultrasound has been and continues to be useful in research investigations, medical applications, and industrial processes, and it is expected that increasing the available intensity will make possible a broader scope of basic research studies and technical applications. Ultrasonic intensities greater than have been realized heretofore from unfocused systems can be produced by composite transducers designed as described in this paper. In the field of biophysics, intensities in the range from approximately $(10)^2$ to $(10)^4$ w/cm^2 are used to produce unique changes in tissue structures of complex biological systems.[5,8] In some cases, where it is desired to produce high intensities

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in relatively small tissue volumes in deep structures without affecting intervening tissue, lens systems are employed to focus the acoustic energy. Some of the present designs employ a layered structure composed of a piezoelectric element, a single coupling medium, and a lens; the essential components of the transducer which produces and couples the acoustic energy to the transmitting medium. The technical literature contains design criteria for such transducers which specify that the specific acoustic impedance of the lens material be as nearly equal to that of the propagating medium as possible and that the thickness of the single coupling medium be as thin as possible.[6] An objective of this paper is the demonstration that the choice of conditions usually stated in the literature to realize "optimum" performance from composite transducers is neither necessary nor desirable, that is, it is shown that other choices of parameters and dimensions enjoy distinct advantages over those usually prescribed.

II. ANALYSIS

The type of system considered here is illustrated in Fig. 1. One face of a piezoelectric element e is coupled to a medium r, in which acoustic radiation is desired, by the planar layers of spacing medium s and coupling medium p. The characteristic acoustic impedances of these media are, in general, all different. It is assumed that the piezoelectric element is operated at its fundamental resonant frequency, at which it is a half-wavelength thick (or an odd harmonic in which case it is an odd number of half-wavelengths thick), that it is terminated on the opposite face by a material of zero acoustic impedance, and that all media are acoustically lossless. The analysis proceeds by computing the motional part of the electrical input impedance at the electrical terminals of the piezoelectric element. This impedance is dependent upon the acoustic input impedance into medium s, i.e., Z_s of the figure. The acoustic intensity I_{rc} into medium r can then be expressed in terms of the real part of the electrical impedance function as follows:

$$I_{rc}=4e_{ik}^2E^2\,\mathrm{Re}(1/Z_s), \tag{1}$$

where e_{ik} designates the appropriate piezoelectric stress constant for the transducer material (e_{11} in the case of an X-cut quartz plate) and E is the amplitude of the electric field strength.

The acoustic impedances Z_r, Z_p, and Z_s, looking to the right at the boundaries indicated in Fig. 1 are, respectively,

$$Z_r=R_r, \tag{2}$$

$$Z_p=R_p\left[\frac{Z_r+jR_p\tan(\omega L_p/v_p)}{R_p+jZ_r\tan(\omega L_p/v_p)}\right], \tag{3}$$

and

$$Z_s=R_s\left[\frac{Z_p+jR_s\tan(\omega L_s/v_s)}{R_s+jZ_p\tan(\omega L_s/v_s)}\right]. \tag{4}$$

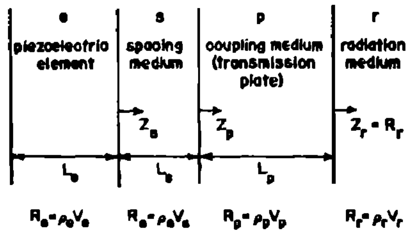

FIG. 1. Schematic diagram of composite transducer.

These formulas, together with relation (1), constitute the necessary analytic expressions from which the intensity gain of the composite system over that of a system in which the piezoelectric element radiates directly into medium r compared for equal values of the driving electric field strength, can be computed. For the direct radiation case, the intensity I_{rd} is

$$I_{rd}=4e_{ik}^2E^2/R_r. \tag{5}$$

Rather than express the intensity gain in the general case by elimination of the appropriate variables from the above relations, it is more appropriate to consider and interpret a series of special cases.

Case 1

Let

$$L_s=\tfrac{1}{4}\lambda_s. \tag{6}$$

In this case Z_s is

$$Z_s=\frac{R_s^2}{R_p}\left\{\frac{\dfrac{R_p}{R_r}\left[1+\tan^2\dfrac{\omega L_p}{v_p}\right]+j\left[1-\left(\dfrac{R_p}{R_r}\right)^2\right]\tan\dfrac{\omega L_p}{v_p}}{1+\left(\dfrac{R_p}{R_r}\right)^2\tan^2\dfrac{\omega L_p}{v_p}}\right\}. \tag{7}$$

Upon substituting (7) into (1), the expression for the acoustic intensity radiated into medium r is

$$I_c=\frac{4e_{ik}^2E^2}{R_r}\left(\frac{R_p}{R_s}\right)^2\left[\frac{1+\tan^2(\omega L_p/v_p)}{(R_p/R_r)^2+\tan^2(\omega L_p/v_p)}\right]. \tag{8}$$

The gain, $G(\lambda_s/4)$ (the gain for $L_s=\frac{1}{4}\lambda_s$) in intensity, for equal values of the driving electric field, of the composite system over the direct drive system is obtained by forming the ratio of Eq. (8) to Eq. (5).

$$G(\tfrac{1}{4}\lambda_s)=\left(\frac{R_r}{R_s}\right)^2\left[\frac{1+\tan^2(\omega L_p/v_p)}{1+(R_r/R_p)^2\tan^2(\omega L_p/v_p)}\right]. \tag{9}$$

The bracketed part of this formula is illustrated graphically in Fig. 2. The gain is shown as a function of the quantity L_p/λ_p with the parameter from curve to curve being the ratio of the characteristic impedances of media p and r. It is apparent that the gain of the system increases as the characteristic impedance of medium p (the transmission plate) increases, that is, as R_p becomes larger relative to R_r. The maximum gain for a

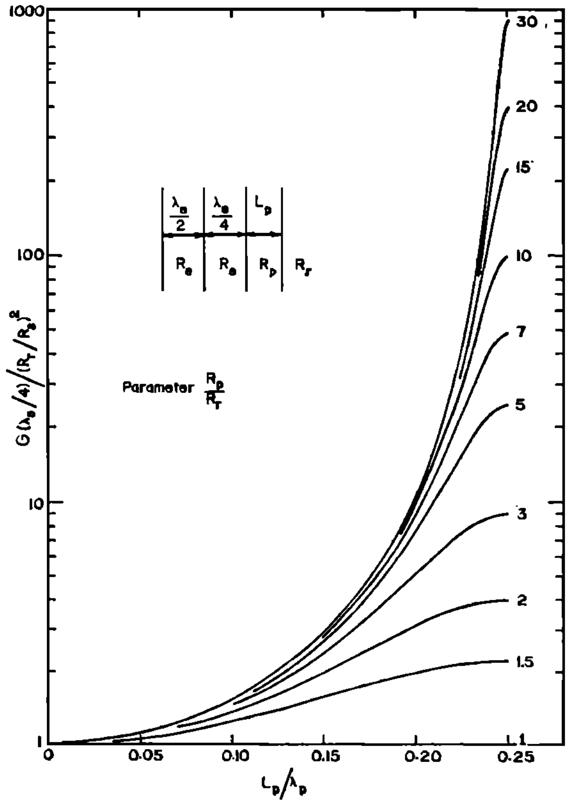

FIG. 2. Gain of composite transducer for $\frac{1}{4}$ wavelength of spacing material vs thickness of transmission plate. Symmetrical about $L_p/\lambda_p=0.25$ and repeats every $L_p/\lambda_p=0.5$.

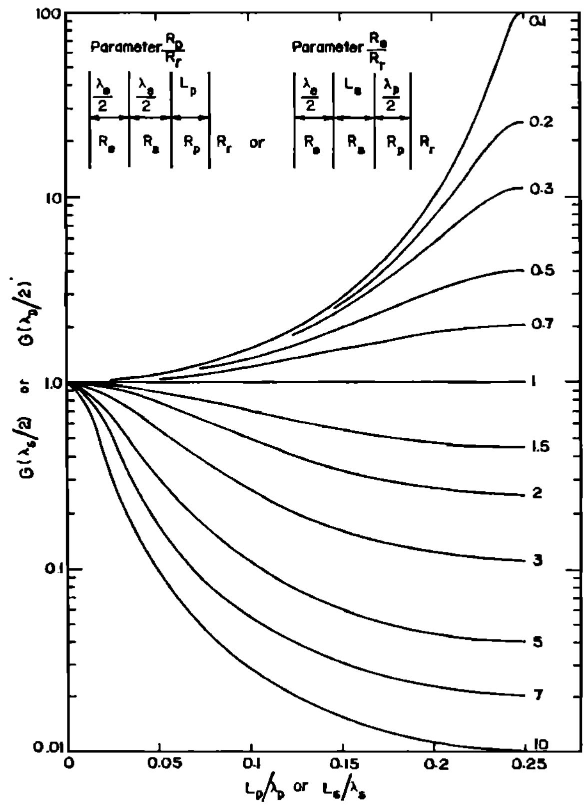

FIG. 3. Gain of composite transducer for $\frac{1}{2}$ wavelength of spacing material vs thickness of transmission plate and for $\frac{1}{2}$ wavelength of transmission plate vs thickness of spacing material, respectively. Symmetrical about L_p/λ_p, $L_s/\lambda_s=0.25$ and repeats every L_p/λ_p, $L_s/\lambda_s=0.5$, respectively.

specific choice of material is achieved when the thickness L_p of the transmission plate is equal to one quarter-wavelength or an odd multiple thereof.

A further increase in output intensity can be achieved for a constant value of the driving electric field by choosing a material for the coupling spacer with a characteristic acoustic impedance R_s smaller than the characteristic impedance of the medium into which the acoustic energy radiates. The values of the gain from the graph of Fig. 2 are directly multiplicable by the ratio $(R_r/R_s)^2$ of these characteristic impedances.

It should be noted that the gain in intensity of the composite system over that characteristic of a system with the same piezoelectric element radiating directly into the medium, r, is achieved as the result of the reduction of the electrical input impedance (the motional branch) into the piezoelectric element. This decrease in electrical input impedance results from the reduced mechanical impedance at the face of the piezoelectric element which is produced by the composite coupling assembly, thereby enabling the element to draw more electric current and consequently more power at the same driving voltage.

Case 2

Let

$$L_s=\tfrac{1}{2}\lambda_s. \tag{10}$$

Then Z_s is

$$Z_s=R_p\left\{\frac{\left(\frac{R_p}{R_r}\right)\left[1+\tan^2\frac{\omega L_p}{v_p}\right]+j\left[\left(\frac{R_p}{R_r}\right)^2-1\right]\tan\frac{\omega L_p}{v_p}}{\left(\frac{R_p}{R_r}\right)^2+\tan^2\frac{\omega L_p}{v_p}}\right\}, \tag{11}$$

and the radiated intensity is

$$I_c=\frac{4e_{ik}^2E^2}{R_r}\left[\frac{1+\tan^2(\omega L_p/v_p)}{1+(R_p/R_r)^2\tan^2(\omega L_p/v_p)}\right]. \tag{12}$$

The gain $G(\frac{1}{2}\lambda_s)$ in this case is

$$G(\tfrac{1}{2}\lambda_s)=\frac{1+\tan^2(\omega L_p/v_p)}{1+(R_p/R_r)^2\tan^2(\omega L_p/v_p)}. \tag{13}$$

This expression is represented in graphical form in Fig. 3, where the gain is exhibited as a function of the parameters L_p/λ_p and the ratio R_p/R_r of the characteristic impedances. It is seen that the gain is less than unity for all values of the ratio $R_p/R_r>1$ and greater than unity for all values of $R_p/R_r<1$.

Case 3

Let

$$L_p = \tfrac{1}{4}\lambda_p. \tag{14}$$

Then Z_s is

$$Z_s = R_s \left\{ \frac{\left(\frac{R_p}{R_r}\right)\left(\frac{R_p}{R_s}\right)\left(1+\tan^2\frac{\omega L_s}{v_s}\right) + j\left[1-\left(\frac{R_p}{R_r}\right)^2\left(\frac{R_p}{R_s}\right)^2\right]\tan\frac{\omega L_s}{v_s}}{1+\left(\frac{R_p}{R_r}\right)^2\left(\frac{R_p}{R_s}\right)^2\tan^2\frac{\omega L_s}{v_s}} \right\}, \tag{15}$$

and the radiated intensity is

$$I_c = \frac{4e_{ik}^2E^2}{R_r}\left[\frac{\left(\frac{R_p}{R_s}\right)^2\left(1+\tan^2\frac{\omega L_s}{v_s}\right)}{\left(\frac{R_p}{R_r}\right)^2\left(\frac{R_p}{R_s}\right)^2+\tan^2\frac{\omega L_s}{v_s}}\right]. \tag{16}$$

The gain, $G(\frac{1}{4}\lambda_p)$, in this case is

$$G(\tfrac{1}{4}\lambda_p) = \frac{1+\tan^2(\omega L_s/v_s)}{\left(\frac{R_p}{R_r}\right)^2+\left[\left(\frac{R_s}{R_r}\right)^2\Big/\left(\frac{R_p}{R_r}\right)^2\right]\tan^2\frac{\omega L_s}{v_s}}. \tag{17}$$

This expression is illustrated in graphical form in Fig. 4, where the gain is plotted as a function of the parameter L_s/λ_s for various values of the ratio R_p/R_r for the case $R_s/R_r = 1$. These curves show that the gain can be greater than, equal to, or less than unity, depending upon the thickness of the spacing medium in wavelengths, for any fixed values of the two ratios of characteristic impedances. The effect of varying the value of the ratio R_s/R_r for a fixed value of the ratio R_p/R_r is to alter drastically the gain for values of L_s near $\frac{1}{4}\lambda_s$ and not to produce significant alterations for values deviating significantly from $L_s = \frac{1}{4}\lambda_s$. Equation (17) shows that the gain is increased for $R_s/R_r < 1$ and decreased for $R_s/R_r > 1$.

Case 4

Let

$$L_p = \tfrac{1}{2}\lambda_p. \tag{18}$$

Then Z_s is

$$Z_s = \frac{R_s\left(\frac{R_s}{R_r}\right)\left(1+\tan^2\frac{\omega L_s}{v_s}\right)+j\left[\left(\frac{R_s}{R_r}\right)^2-1\right]\tan\frac{\omega L_s}{v_s}}{(R_s/R_r)^2+\tan^2(\omega L_s/v_s)}, \tag{19}$$

and the radiated intensity is

$$I_c = \frac{4e_{ik}^2E^2}{R_r}\left[\frac{1+\tan^2(\omega L_s/v_s)}{1+(R_s/R_r)^2\tan^2(\omega L_s/v_s)}\right]. \tag{20}$$

The gain, $G(\frac{1}{2}\lambda_p)$ in this case is

$$G(\tfrac{1}{2}\lambda_p) = \frac{1+\tan^2(\omega L_s/v_s)}{1+(R_s/R_r)^2\tan^2(\omega L_s/v_s)}. \tag{21}$$

This expression is identical in form to Eq. (13) and is also illustrated graphically in Fig. 3 where the gain, $G(\lambda_p/2)$, is plotted as a function of the parameter L_s/λ_s and the ratio, R_s/R_r, of characteristic impedances.

III. DISCUSSION

It is apparent from the analysis and graphs of the preceding section that the radiated acoustic power per unit area of a composite assembly, consisting of a piezoelectric element coupled through "spacing" and

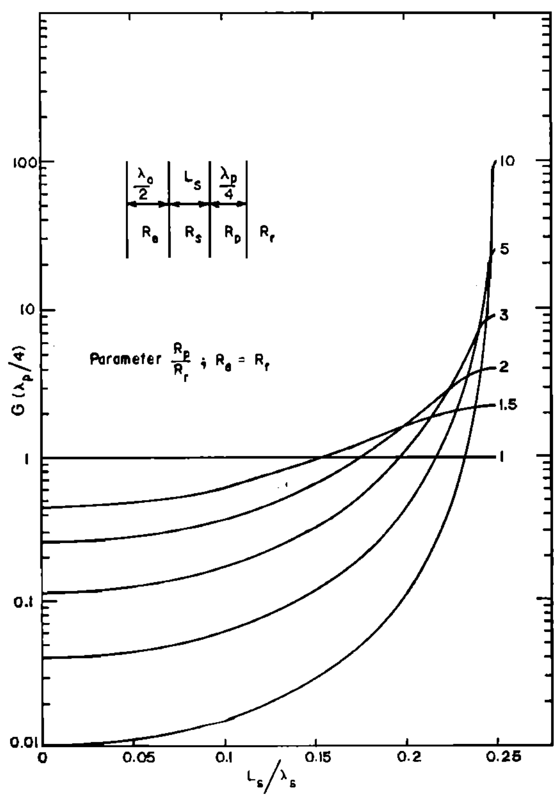

FIG. 4. Gain of composite transducer for $\frac{1}{4}$ wavelength of transmission plate vs thickness of spacing material for case $R_s/R_r = 1$. Symmetrical about $L_s/\lambda_s = 0.25$ and repeats every $L_s/\lambda_s = 0.5$.

"transmission" media (see Fig. 1), can be either greater or less than that from a transducer with the piezoelectric element radiating directly into the radiation medium, for equal values of the electric field strength.

The ratio of these intensities, which is defined in this paper as the *gain* of the composite transducer, can be varied over a wide range of values by appropriate choice of the dimensions and characteristic impedances of the spacing and transmitting media. Thus, the maximum intensities available into liquids at the transducer face at maximum driving electric field strength are considerably increased. This is to be compared with the gain obtained by inserting a single quarter-wave layer having the root-mean-square characteristic impedance of the piezoelectric element and the radiation medium between these two media. This gain, which is $(R_r/R_s)^2$, depends upon the characteristic impedance of the radiation medium, whereas in the case of two quarter-wave layers, the gain is $(R_p/R_s)^2$ and depends upon the characteristic impedance ratio of the coupling material to that of the spacing material. It is of interest to compare the gain realized by employing two quarter-wave layers over a single quarter-wave layer, which is $(R_p/R_r)^2$. The analytic method used in this paper can be extended to any number of layers, from which it will be seen that further increase in the intensity gain, for equal values of the applied electric field strength, can be obtained.[4] The dimensions of the elements of a composite transducer are of critical importance especially if high intensity gains are to be achieved. This means that the transducers must be designed to compensate for the effects of temperature changes in the materials when the devices operate "continuously" under high-power conditions, i.e., the dimensions of the coupling layers, in wavelengths, must remain within the necessary tolerances. The gain is, of course, a periodic function of the layer thicknesses, i.e., odd multiples of a quarter-wavelength thickness yield identical values of the gain. However, if temperature effects are to be minimized, the choice of thickness for many applications would be one-quarter wavelength since the thinner the coupling layers, the smaller will be the effects produced by temperature changes in the system.

From an examination of the graphs, it is apparent that intensity gains in water (or liquids in general) in the range from 100 to 1000 can be achieved. For example, if a material with a characteristic impedance approximately equal to that of water is chosen for the spacing element and a material such as stainless steel with a characteristic impedance approximately 30 times that of water is used for the transmission plate, then for one-quarter wavelength thicknesses of these materials the gain of the composite system compared with the case of direct radiation of the piezoelectric element into water is nearly 1000.

Composite transducers of the type considered here, when combined with appropriate lens systems, will make possible the attainment of ultra-high intensity acoustic beams, i.e., intensities as great as $(10)^7$ to $(10)^8$ w/cm^2.