Crystal Acoustic Arrays

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Citation: The Journal of the Acoustical Society of America **25**, 336 (1953); doi: 10.1121/1.1907045
View online: http://dx.doi.org/10.1121/1.1907045
View Table of Contents: http://asa.scitation.org/toc/jas/25/2
Published by the Acoustical Society of America

the conversion of sound energy into thermal energy in a definite ratio J which is the same as the Joule's equivalent.

A detailed paper will be published in due course.

[1] Papers in course of publication.
[2] Parthasarathy, Srinivasan, and Chari, Nature 166, 828 (1950), and others in course of publication.

Attenuation of Sound in a Tube

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(Received December 29, 1952)

An experimental method of measuring the attenuation of sound in a circular tube for gases at reduced pressures has been developed. Since the tube effect is inversely proportional to the square root of the pressure, a reduction in pressure increases the attenuation and thus increases the experimental accuracy. The tube attenuation was measured for the frequency over pressure ratio of 8 to 2000 kc/atm. The measured attenuation was found to be 4.5 percent greater than that predicted by the Kirchhoff formula.

FOR many years now, there has existed an uncertainty in the magnitude of the attenuation of sound produced by circular tubes. An experimental method for measuring the tube effect for gases at reduced pressures has been developed.

Kirchhoff's formula for the viscous and thermal losses at the walls of a tube as given by Rayleigh is

$$\alpha_\tau=(\omega/2)^{\frac{1}{2}}\gamma'/ar, \tag{1}$$

where $\gamma'=\mu'^{\frac{1}{2}}+(a/b-b/a)v^{\frac{1}{2}}$; $a=(\gamma p/\rho)^{\frac{1}{2}}$ adiabatic sound velocity; $b=(p/\rho)^{\frac{1}{2}}$ isothermal sound velocity; $\mu'=\mu/\rho$ kinematic shear viscosity; $v=K/C_v\rho$ thermometric conductivity; $K=$thermal conductivity; $C_v=$specific heat at constant volume; $\rho=$density of gas; $\omega=$angular frequency of the sound; $r=$radius of the tube.

Equation (1) has been investigated on numerous occasions. Most investigators agree with the Kirchhoff formula as to radius and frequency dependence; however, there is a wide variation as to the actual magnitude of the attenuation. Equation (1) may be written in the form

$$\alpha_\tau=(\pi\mu)^{\frac{1}{2}}\left[\frac{1}{\gamma^{\frac{1}{2}}}+\frac{\gamma-1}{\gamma}\left(\frac{K}{C_v\mu}\right)^{\frac{1}{2}}\right]\frac{1}{r}\left(\frac{f}{p}\right)^{\frac{1}{2}}. \tag{2}$$

It is apparent from this expression that both frequency and pressure may be used as variables for a fixed gas and tube radius. The use of the pressure variable enables one to increase the magnitude of the attenuation to be measured and thus increase the experimental accuracy.

An investigation of the tube attenuation has been made for the frequency range of 2 to 10 kc and for pressures ranging from 2 to 250 mm of Hg. The sound tube was made of glass tubing of 0.841 cm radius and approximately 120 cm long. A small ribbon speaker, which floated on tefelon rings and was capable of being moved along the axis of the tube, was used as a sound source. A condenser microphone, mounted with its diaphragm flush with the tubing wall, was used for a sound pick-up. A sound pressure recorder was employed to measure the sound pressure as the distance between the microphone and the speaker was varied. The attenuation coefficient was then determined from the slope of this record. The sound tube and accompanying gas handling equipment was made vacuum tight. The system could be evacuated to a pressure of 10^{-6} mm of Hg.

The attenuation coefficient was measured for both dry air and dry nitrogen for the frequency over pressure ratio of from 8 to 2000 kc/atmos. The measured attenuations were corrected for the classical gas attenuation and then plotted against the ratio of frequency over pressure. The best straight line was drawn through the experimental points for each gas. The resulting curve may be expressed in the form

$$\alpha_T=K_1(f/p)^{\frac{1}{2}}\ \text{db/cm}, \tag{3}$$

where K_1 is the experimental coefficient, f the frequency in kc, and $p=$pressure in mm of Hg. This expression may now be compared with the attenuation predicted by Eq. (2). The comparison is most easily made if one replaces K_1 by a theoretical coefficient K_2 which is determined by applying the appropriate constants for each gas in Eq. (2). The results for two runs on dry air and dry nitrogen at 23°C are shown in Table I. The results of this investigation show:

TABLE I. Coefficients for Eq. (3).

Gas	K_1	K_2	% difference
Air	0.274	0.263	4.2
Air	0.276	0.263	4.9
Nitrogen	0.269	0.259	3.8
Nitrogen	0.272	0.259	4.9

1. The Kirchhoff formula is correct as to the one-half power dependence of the ratio of frequency over pressure.

2. The Kirchhoff formula predicts an attenuation which is 4.5 percent lower than observed. The actual attenuation for a glass tube may be determined by applying the experimentally determined factor 1.045 to Eq. (2).

Crystal Acoustic Arrays*

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(Received December 20, 1952)

A point source method for rapid design of a crystal transducer array using three crystal thicknesses in the ratio of 1:2:4 has been developed. Formulas are presented for main lobe width and ratio of secondary lobe amplitude to main lobe amplitude. For a 10×10 array (100 elements) this method of design leads to a relative side lobe level of 22 db as compared to a value for a uniform array of 13.4 db at the expense of having a beam width 1.25 times as wide as the uniform array.

IN analyzing multi-element acoustic arrays to obtain beam patterns of electroacoustic transducers two common approaches are used. One is to consider the transducer as a whole with a certain continuous variation of velocity amplitude over its face and to compute the beam pattern from the appropriate integrals.[1,2] The other approach is to consider the array as made up of point sources located at the center of the elements, compute the beam pattern from the appropriate summation, and then modify it with the directional pattern of the elements.[3,4] The latter approach is often preferred since in many arrays the elements are quite discontinuously placed, and more important, if properly handled it results in simple formulas for computing the two important properties, main lobe width and ratio of amplitudes of secondary lobes to the amplitude of the main lobe. The most sophisticated application of this second approach appears to be that of Davids, Thurston, and Mueser.[4] In applying electromagnetic antenna analysis of linear arrays to the problem of designing electroacoustic transducers, they made use of the Tschebyscheff polynomial optimum type of synthesis. Their method is admirably suited to the design of magnetostrictive transducers in which it is a fairly simple matter to adjust for the many different field strengths required—in their case seventeen. In designing a crystal element transducer it is preferable to reduce this number greatly since it might require the use of seventeen different crystal thicknesses. For some types of crystal transducer construction it is not feasible to use such a large number of different sizes. The extreme in simplicity would be a uniform array with all crystals the same size. This design has the unfortunate concomitant of very high amplitude side lobe levels. The purpose of this letter is to present a design method based on linear array analysis in which only three crystal thicknesses are needed, in the proportion of 1:2:4 and which has considerably better side lobe suppression than a uniform array although not as good suppression as the optimum Tschebyscheff array.

The procedure used is based on the Schelkunoff method of linear array analysis.[5] Formulas especially suited to rapid com-

putation are developed for main lobe width and relative side lobe level.

If

$$\psi=\beta d\cos\theta,$$

where d=distance between centers of array elements, $d<\lambda$ to avoid repetition of major lobes, θ=angle measured from the plane of the array, $\beta=2\pi/\lambda$, and if $z=e^{i\psi}$, then the relative pressure amplitude pattern far from the source for a linear array of uniformly spaced elements 0, 1, 2, $\cdots$ $(n-1)$ is given by

$$|P|=|A_0+A_1z+A_2z^2+\cdots+A_{n-1}z^{n-1}|, \tag{1}$$

where A_i is proportional to the driving electrical field strength of the ith crystal. If a square crystal matrix of $n\times n$ blocks is considered, then the corresponding linear array to be synthesized is an n element one.

For the method of design under consideration the polynomial corresponding to Eq. (1) is formed as follows:

$$|P|=P_0|(1+z+\cdots+z^{n-4}+z^{n-3})(1+z^2)|. \tag{2}$$

This can be expanded so that

$$|P|=P_0|1+z+2z^2+2z^3+\cdots+2z^{n-3}+z^{n-2}+z^{n-1}|. \tag{3}$$

The amplitude can be expressed as

$$|P|=P_0|(1+z^2)(1-z^{n-2})/(1-z)| \tag{4}$$

or

$$|P|=P_0\left|\left(\sin\frac{n-2}{2}\psi/\sin\frac{\psi}{2}\right)\cdot 2\cos\psi\right|. \tag{5}$$

The nulls of this pattern occur when

$$(n-2)/2\cdot\psi=k\pi \quad k=1, 2,\cdots \tag{6}$$

and

$$\psi=(2m+1)\pi/2 \quad m=0, 1. \tag{7}$$

The first null after the main lobe occurs when

$$\psi=2\pi/(n-2). \tag{8}$$

We are interested in this from the point of view of the spatial angle; therefore if the first null occurs at an angle $(\pi/2+\Delta\varphi)$, where $\Delta\varphi$ is the half-width of the main lobe, then[5]

$$\cos(\pi/2+\Delta\varphi)=\lambda/(n-2)d. \tag{9}$$

If $\Delta\varphi$ is small, $\Delta\varphi\approx\lambda/(n-2)d$ or the beam width of the main lobe is

$$2\Delta\varphi=2\lambda/(n-2)d. \tag{10}$$

This beam width is the same as that which is obtained from a uniform array of $(n-2)$ elements.

In order to obtain the side lobe level we can refer back to Eq. (5). At the maximum of the main lobe ($\psi=0$), $|P|=2(n-2)P_0$. At the first side lobe maximum ($\psi\approx 3\pi/(n-2)$), for large n

$$|P|=0.212\times 2(n-2)P_0\cos 3\pi/(n-2). \tag{11}$$

The ratio of secondary lobe amplitude to main lobe amplitude is thus

$$0.212\cos 3\pi/(n-2). \tag{12}$$

For $n=10$, as an example, this ratio is 0.082, and the side lobe is down 22 db. For a uniform array of 10 elements this value would be about 13.4 db. This ratio is further improved if the directional pattern of the crystal elements is considered.

Equation (3) can be used to get the relative field strengths from the coefficients. These are all in the ratio of 1:1 or 1:2. In order to get the values for a two-dimensional array the product theorem of Davids, Thurston, and Mueser[4] can be used. It is apparent that we shall then get field strengths in the ratio of 1:2:4 or crystal thicknesses in the same proportions.

In conclusion, by using the point source method of analysis a basis for rapid design of a crystal transducer array using a small number of simply related crystal thicknesses has been presented. For a 10×10 array (100 elements) a relative side lobe level of 22 db is obtained as compared to a uniform array value of 13.4 db, at the expense of having a beam width 1.25 times as wide as the uniform array. The beam width is given by

$$2\Delta\varphi=2\lambda/(n-2)d, \tag{10}$$

where $2\Delta\varphi$ is the main lobe width and d is the distance between centers of array elements.

The ratio of secondary lobe amplitude to main lobe amplitude, r, is given by

$$r=0.212\cos(3\pi/(n-2)). \tag{12}$$

* This work was supported by the Bureau of Ships under Contract Nobsr-52367.

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[2] R. Clark-Jones, J. Acoust. Soc. Am. 16, 147 (1945).
[3] J. Wolff and L. Malter, J. Acoust. Soc. Am. 2, 201 (1930).
[4] Davids, Thurston, Mueser, J. Acoust. Soc. Am. 24, 50 (1952).
[5] See, for example, E. C. Jordan, *Electromagnetic Waves and Radiating Systems* (Prentice-Hall, Inc., New York, 1950).

Symposium on Ultrasonic Absorption in Fluids at Brown University (October, 1952)

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(Received December 24, 1952)

THIS is a brief report on a symposium on ultrasonic absorption in fluids held in the Ultrasonics Laboratory of Brown University on October 16 and 17, 1952. Sponsored by the U. S. Office of Naval Research, it was attended by about seventy-five acoustical physicists, and fifteen papers were presented in four technical sessions. Most of these provoked extensive, vigorous, and profitable discussion. On the evening of October 16 the various research set-ups of the Ultrasonics Laboratory were open for inspection.

H. O. Kneser of the Technische Hochschule, Stuttgart, led off with an interesting review of the general theory of absorption in fluids. Using general considerations of irreversible thermodynamics, following closely the method of Meixner, he showed how a general equation of state involving p, $\dot{p}$, s, and $\dot{s}$ could be derived. From this a relaxation absorption coefficient follows without recourse to a specific mechanism. There was some discussion as to the theoretical meaning and possible determination of the arbitrary thermodynamical parameters appearing in the state equation. K. F. Herzfeld of the Catholic University of America followed with a discussion of methods for the numerical calculation of relaxation times and also treated the temperature dependence of absorption. E. A. Hiedemann of Michigan State College presented a unified theory of relaxation phenomena in fluids from the point of view of the general theory of the response of linear systems. This has been applied to the birefringence caused by a longitudinal wave in a liquid and the acoustic behavior of a liquid crystal in a magnetic field. Other papers having a direct relation to the relaxation theory of absorption were given by F. W. deWette of Utrecht (during 1952–53 at Brown University) on the theoretical calculation of acoustical relaxation times in gases by the use of the quantum mechanical theory of molecular collisions; by R. T. Beyer of Brown University on acoustic relaxation in nonideal gases (both a van der Waals gas and a virial gas); and by T. A. Litovitz of Catholic University of America on ultrasonic and dielectric relaxation in glycerol, in which experiments were reported substantiating the assumption that the relaxation times associated with shear and bulk viscosities, respectively, are nearly equal.

A somewhat provocative paper was presented by C. Truesdell of the University of Indiana on the classical hydrodynamic theory of absorption of forced plane infinitesimal waves in pure fluids. The author maintained the thesis that insufficient attention has been paid so far to the classical Stokes-Kirchhoff theory of sound dispersion and absorption as due to viscosity and heat conduction and that by abandoning the well-known Stokes relation and making proper choice of parameters (e.g., the equivalent of the bulk viscosity) considerable agreement with experimental absorp-