

APPENDIX A: REVIEW OF THERMODYNAMICS

This appendix contains a brief review of thermodynamic terminology and the 1st Law of Thermodynamics. Often, the fields of acoustics and thermodynamics are studied by different people. As a result, the two disciplines develop different methods of describing the same phenomenon. In this appendix, the basic principles of thermodynamics will be restated in an acoustics framework to enhance further derivations.

Review of Basic Terms:

Before discussing the laws governing thermodynamics, it is necessary to review the terminology used to describe the basic physics involved. When discussing thermodynamics, one often speaks of the *state* of the medium [Cengel and Boles, 1994]. The state refers to a unique set of *properties* that capture the behavior of the molecules of interest [Cengel and Boles, 1994]. The properties can either be *intensive*, independent of the size of the medium being considered (i.e., temperature, pressure, entropy, and density), or *extensive*, dependent on the size of the medium (i.e., mass and volume) [Cengel and Boles, 1994]. However, the complete thermodynamic state is known once any two independent intensive properties of the material have been specified provided that the region of interest is a *simple compressible system* [Cengel and Boles, 1994]. A simple compressible system exists whenever electric, magnetic, gravitational, motion, and surface tension effects can be ignored [Cengel and Boles, 1994]. Hence, acoustics almost always deals with simple compressible systems with the only exception being surface tension effects at the boundary of gas/fluid media [Pierce, 1991].

Given these concepts, thermodynamics is nothing more than the study of how a material changes from one thermodynamic state to another. When all of the molecules in a particular region have the same properties, the region is said to be in thermodynamic *equilibrium* [Cengel and Boles, 1994]. A region must be in equilibrium before the state can be specified by its intensive properties. However, it would be convenient if the properties could also be used to describe the intermediate states when the system is in transition. Therefore, it is often assumed that the transition between thermodynamic states is slow enough so that all of the molecules in a region of interest have the same properties at all times. Such a transition is called a *quasi-*

equilibrium process [Cengel and Boles, 1994]. In acoustics, this would mean that the frequency of the sound propagation must be less than some threshold value [Pierce, 1991]. The properties of a material undergoing a quasi-equilibrium process change according to the material's *equation of state* [Cengel and Boles, 1994]. The equation of state governs the relationship between the different properties. The most common equation of state is the ideal gas law [Cengel and Boles, 1994]. For most materials, however, the equation of state is too complicated to be found empirically.

There are two thermodynamic state transitions that are of importance in the study of acoustics: *isothermal* changes and *adiabatic* changes. Under isothermal conditions, the temperature of the region of interest remains constant over time [Cengel and Boles, 1994]. Heat may flow, but this change in energy would be expressed by changes in the thermodynamic properties other than temperature. For an adiabatic process, however, there is no heat flow [Cengel and Boles, 1994]. Because heat flow is driven by temperature differentials between a region and its surroundings, this would mean that either all regions of a medium have the same temperature at all times, or that a region is completely insulated from its surroundings [Cengel and Boles, 1994]. Furthermore, because the specific entropy of a region can only be changed by heat or mass flow, an adiabatic process acting on a region in the absence of mass flow would result in constant entropy over time. In either case, if one of the properties of a system is constant over time, then the complete thermodynamic state can be completely specified by only one intensive property.

In the field of acoustics, the study of changes in the thermodynamic state of the medium is generally expressed in terms of changes in the intensive properties of a *fluid particle* [Pierce, 1991]. A fluid particle is an arbitrary clump of molecules in the same quasi-equilibrium thermodynamic state that move together in the medium. As a result, the total mass of the fluid particle would remain constant over time and there would be no mass flow. As a result, the fluid particle would be considered a *control mass* or *closed system* in thermodynamics [Cengel and Boles, 1994]. However, because the density of the particle would be changed by the acoustic disturbance, the total volume of the fluid particle would vary over time.

The other, more general, type of system in thermodynamics is the *control volume* or *open system* [Cengel and Boles, 1994]. However, some authors, such as Kakac and Yener [1985], restrict the use of the term "system" to only refer to closed systems. The control volume is a

selected region that allows both mass flow in and out of the region as well as for changes in the region's volume [Cengel and Boles, 1994; Kakac and Yener, 1985]. In effect, the control mass would just be a special case of the control volume. The most common special case, however, is when the volume of the region is fixed, the dual of the control mass, and is commonly what is meant by the term “control volume” [Cengel and Boles, 1994].

First Law of Thermodynamics

The first law of thermodynamics is nothing more than the conservation of energy [Cengel and Boles, 1994]. Often, however, it is convenient to specify the law in terms of the time rate of change of the energy rather than the total energy present. In this form, the first law for thermo-mechanical interactions is given as

$$\frac{dE}{dt} = \dot{Q}_i + \dot{Q}_e + \dot{W}, \quad (\text{A.1})$$

where E is the total energy in the system, \dot{Q}_i is the rate heat is generated or removed internally, \dot{Q}_e is the rate heat flows into the system, and \dot{W} is the rate work interacts with the system. If we restrict our attention to a single fluid particle, Equation (A.1) can be rewritten as

$$\begin{aligned} \frac{\partial}{\partial t} \iiint_{V^*} \left(\frac{1}{2} \rho \vec{u} \cdot \vec{u} + \rho e + \frac{1}{2} \sum_{i,j=1}^3 (\lambda_L \theta_d \delta_{ij} \xi_{d_i} + 2\mu_L S_{ij} \xi_{d_i}) \right) dV \\ = \iiint_{V^*} \dot{q}_i dV - \iint_{S^*} \dot{\vec{q}} \cdot \hat{n} dS + \iint_{S^*} \sum_{i,j=1}^3 \sigma_{ij} u_i \hat{e}_j \cdot \hat{n} dS, \end{aligned} \quad (\text{A.2})$$

where the total energy term has been expanded to include both the kinetic energy of the moving molecules, the thermodynamic internal energy e , and the stored elastic energy for an isotropic medium. The elastic energy term involves the Lamé constants λ_L and μ_L , the dilatation θ_d , the strain tensor S , and the displacement $\vec{\xi}_d$. Gravitational potential energy has been ignored because this quantity is in general not time varying for acoustic problems. In this equation, \dot{q}_i is the heat generated within the particle, $\dot{\vec{q}}$ is the heat flow across the boundary of the particle, and σ is a tensor representing the external forces acting on the fluid particle. The other parameters in Equation (A.2) are defined as the outward unit normal for the particle \hat{n} , the velocity \vec{u} , and the density ρ .